EEE102 Circuit Theory

Prof PP Acarnley

room E3.24
e-mail: p.p.acarnley@ncl.ac.uk

website: **http://www.staff.ncl.ac.uk/p.p.acarnley/my_teaching.htm**

EEE102 Circuit Theory: Syllabus

Constant (dc) voltages and currents

Conventions for voltage and current, units, Ohm's Law, Kirchoff's Laws and applications to combining resistances in series and parallel, ideal voltage and current sources; technique of circuit analysis using nodal voltages; modelling of real devices with ideal elements, equivalence of voltage and current sources; Thevenin's Theorem, Norton's Theorem; technique of circuit analysis using mesh currents; power, power transfer from a source to a load; linearity, superposition; star-delta and delta-star transformations.

Transient effects in electrical circuits

Energy storage elements: inductance and capacitance, units; types of time-varying excitation in electrical circuits, transients in circuits with a first-order response by analytic solution of a differential equation, exponential rise and decay, time constant in R-C and R-L circuits; initial conditions, effect of initial condition on response, establishing initial and final conditions in higher-order circuits; energy storage in capacitors and inductors.

Circuits with steady-state sinusoidal excitation

Concepts of frequency, angular frequency, phase shift, amplitude, peak, peak-to-peak, and root-mean-square values; introduction to the j operator and its application in circuit analysis; complex impedance, admittance, resistance, reactance, conductance and susceptance; complex impedances of ideal circuit elements, solution of simple circuits by combining impedances in series and parallel; phasor representation of alternating voltages and currents, phasor diagrams as an aid to understanding; general circuit analysis using j notation (nodal voltage and mesh current).

Resonance

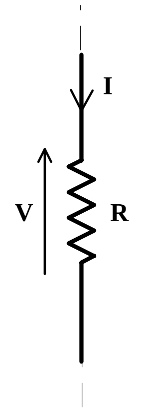
Analysis and applications of series and parallel resonant circuits, bandwidth and Q factor.

Power

Relationships between power, reactive power and VA, power factor, principle of conservation of power and reactive power, reactive power absorbed by capacitors and inductors, power factor correction, complex power in terms of phasor voltages and currents.

DC CIRCUITS

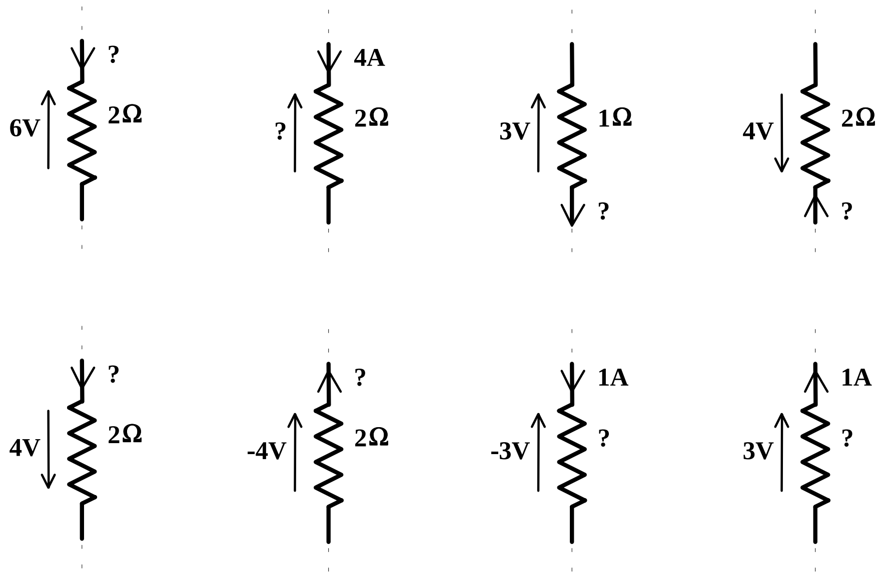
conventions



Ohm's Law: V = R I *with the directions of voltage and current indicated*

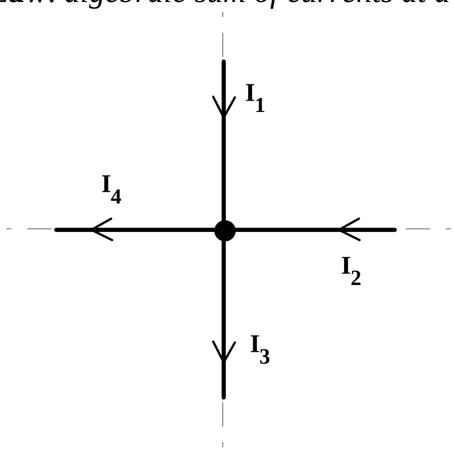
SI UNITS:	quantity	unit
	voltage	Volts (V)
	current	Amps (A)
	resistance	Ohms (Ω)

Ohm's Law: Practice



Kirchoff's Laws

Current Law: *algebraic sum of currents at a node is zero*



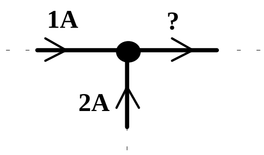
currents into node positive:

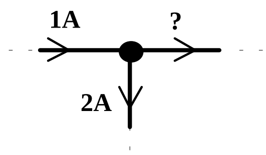
$$I_1 \qquad I_2 \qquad I_3 \qquad I_4 \;=\; 0$$

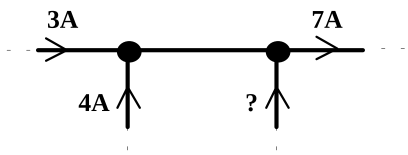
currents into node negative:

$$I_1 \qquad I_2 \qquad I_3 \qquad I_4 \;=\; 0$$

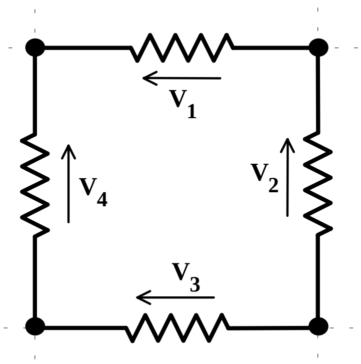
Kirchoff's Current Law: Practice







Voltage Law:
algebraic sum of voltages around a closed circuit loop is zero



clockwise around the loop, against the arrow positive:

$$V_1 \qquad V_2 \qquad V_3 \qquad V_4 \qquad = 0$$

clockwise around the loop, with the arrow positive:

$V_1 \quad V_2 \quad V_3 \quad V_4 \quad = 0$

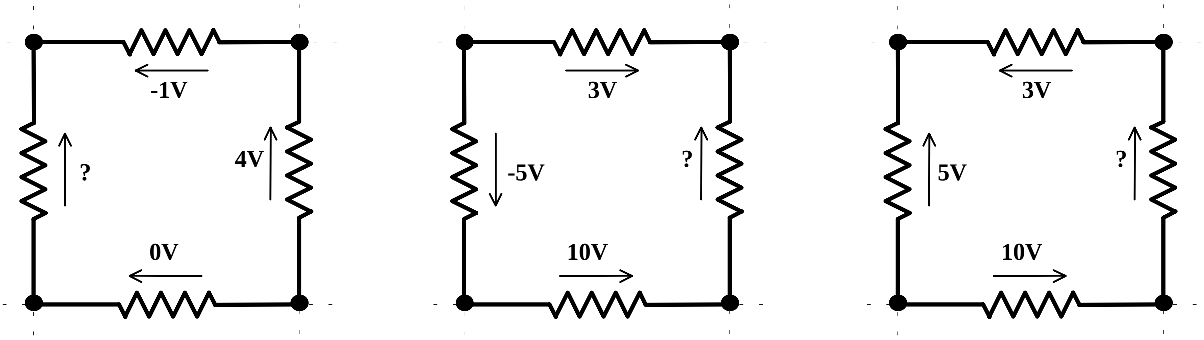
anticlockwise around the loop, against the arrow positive:

$V_1 \quad V_2 \quad V_3 \quad V_4 \quad = 0$

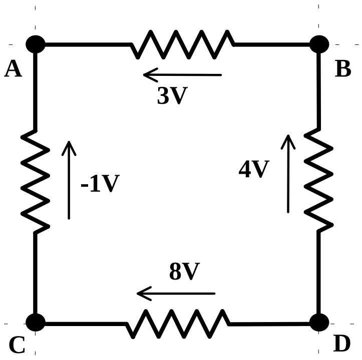
anticlockwise around the loop, with the arrow positive:

$V_1 \quad V_2 \quad V_3 \quad V_4 \quad = 0$

Kirchoff's Voltage Law: Practice



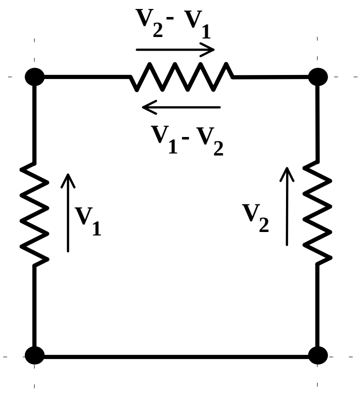
NOTATION



V_{AB} = 3V $\qquad$ V_{BA} = -3V

V_{CD} = $\qquad$ V_{AC} =

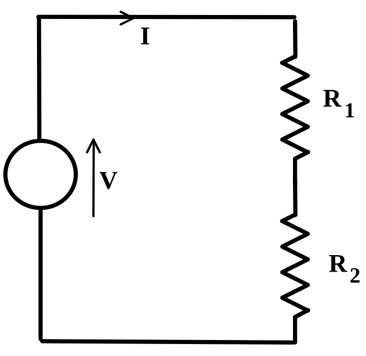
A USEFUL RELATIONSHIP

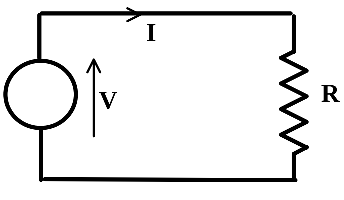


IDEAL VOLTAGE AND CURRENT SOURCES

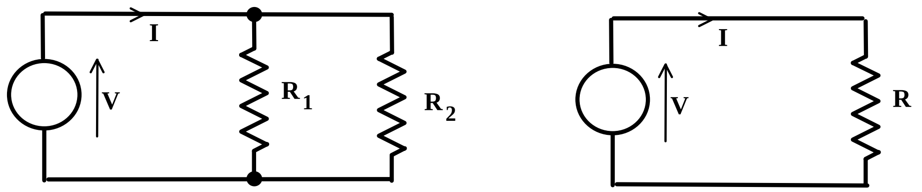
COMBINING RESISTANCES IN SERIES AND PARALLEL

series

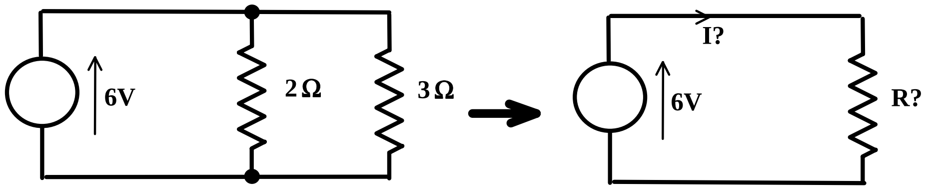


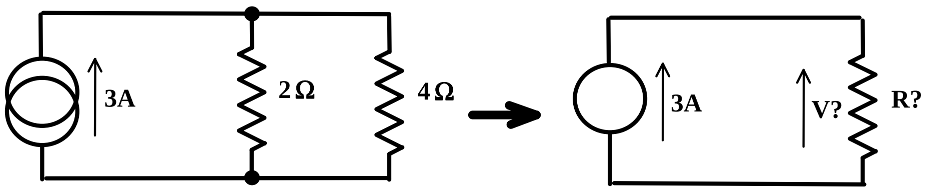


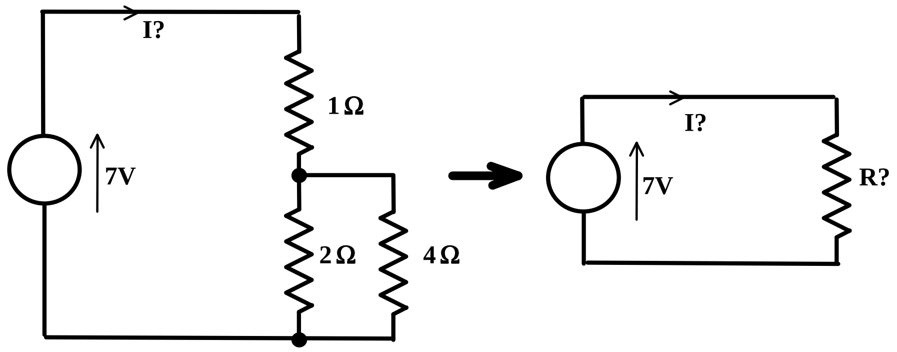
parallel

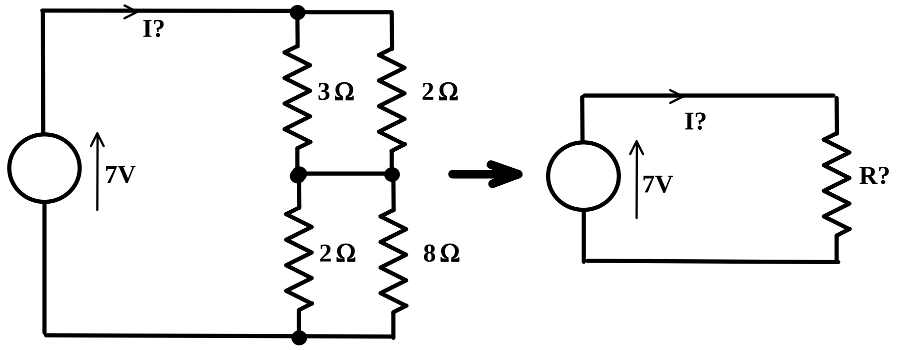


SIMPLIFYING CIRCUITS: PRACTICE

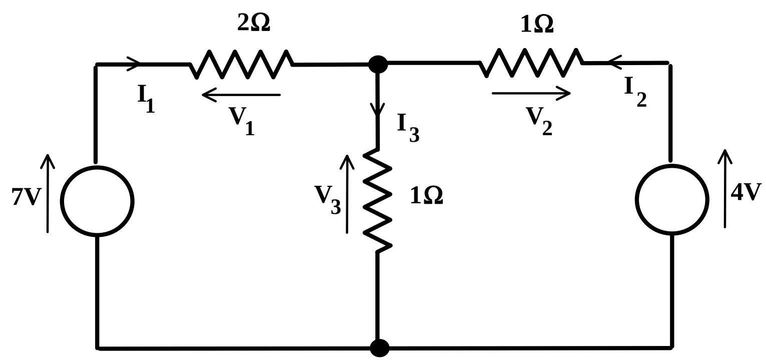




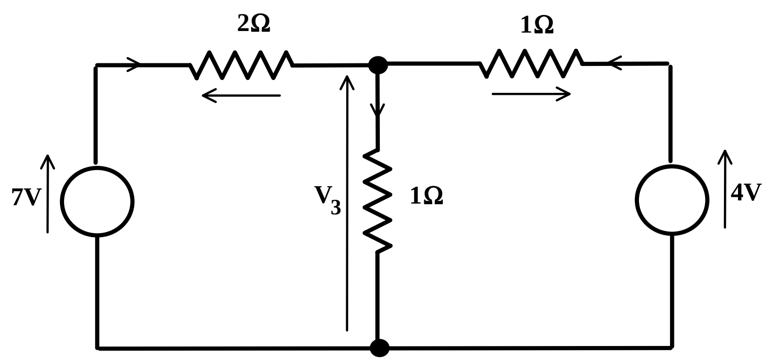




INTRODUCTION TO CIRCUIT ANALYSIS



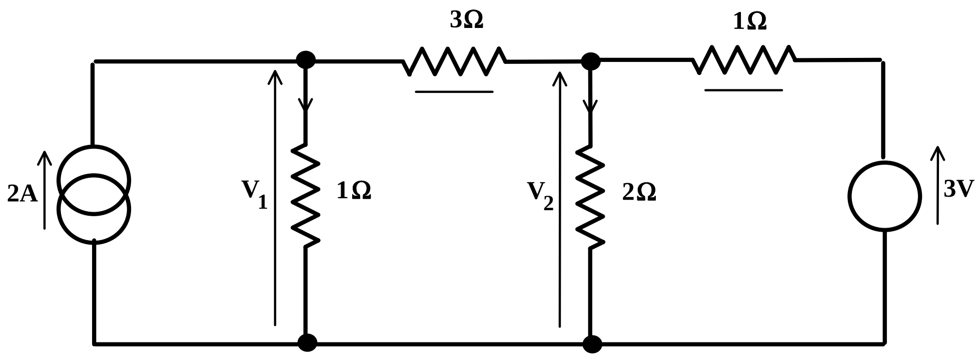
NODAL VOLTAGE ANALYSIS



GENERAL METHOD

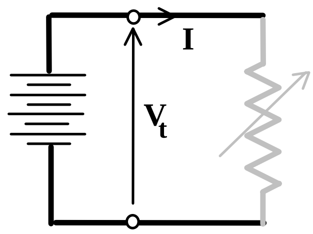
1. Choose a reference node
2. Define voltages at other nodes relative to the reference node
3. Express element voltages in terms of node voltages, using Kirchoff's Voltage Law
4. Calculate element currents using Ohm's Law
5. Write down nodal voltage equations using Kirchoff's Current Law at every node, except the reference node.

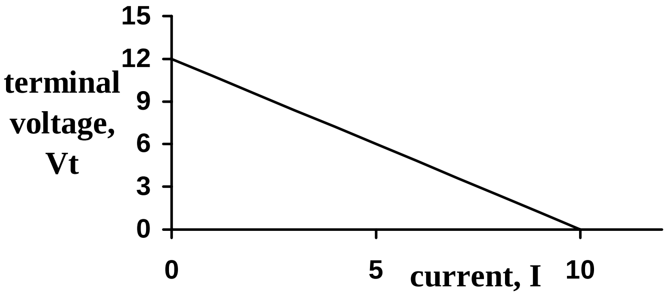
EXAMPLE



MODELLING OF REAL DEVICES WITH IDEAL ELEMENTS

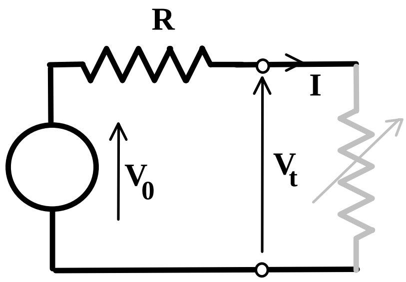
equivalent circuit of a real device, e.g. a battery:



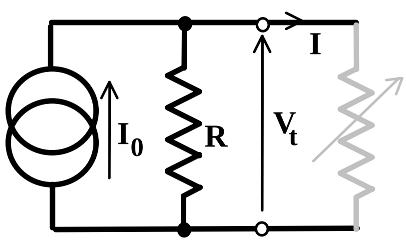


$V_t = V_0 - RI$ (1) $V_t = 12 - 1.2*I$

Relation between terminal voltage and current can be represented by the ‘equivalent circuit’:



Is there another equivalent circuit? Consider:



In this circuit:

$$I_0 - I - (V_t/R) = 0$$

so:

$$RI_0 - RI - V_t = 0$$

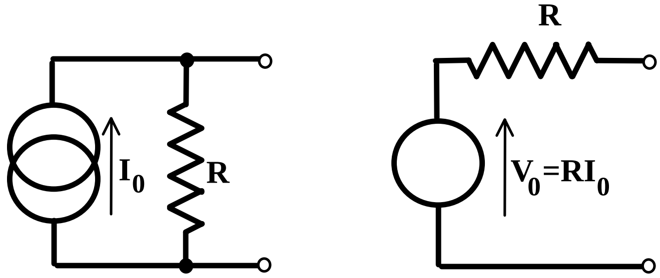
$$V_t = RI_0 - RI \qquad (2)$$

Comparing Equations 1 and 2, we see that the two relationships between terminal voltage and current are identical if:

$V_0 = RI_0$ and if the resistance R in the two equivalent circuits is the same.

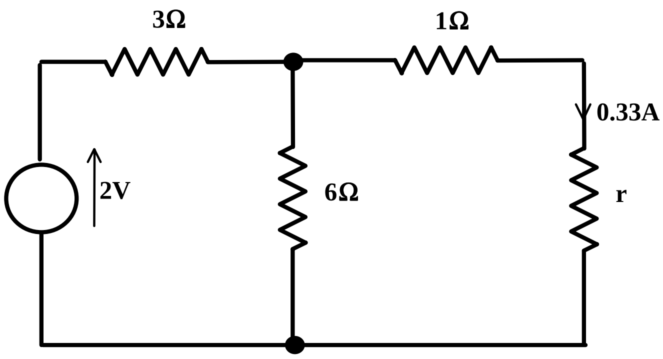
This leads to the idea of:

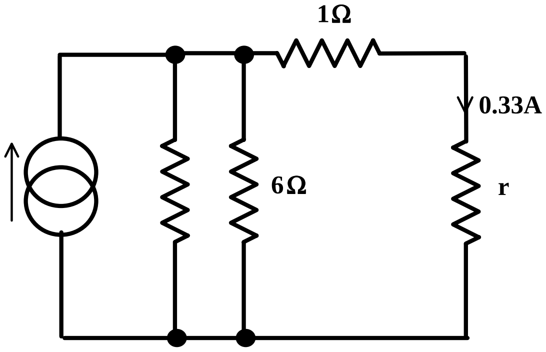
SOURCE EQUIVALENCE

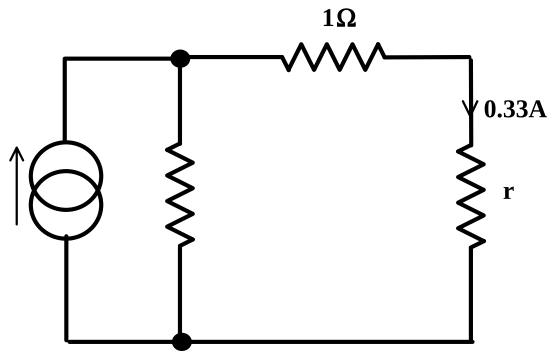


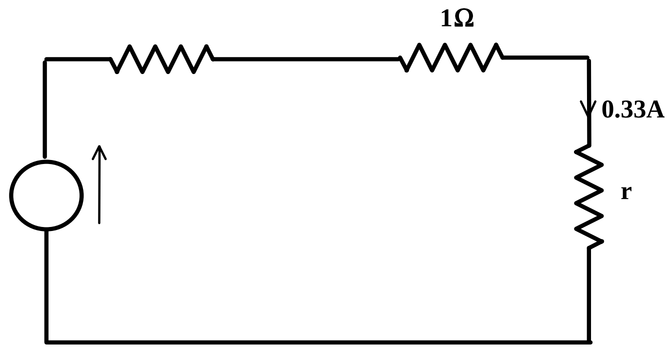
These two circuits have exactly the same terminal behaviour.

Application of Source Equivalence






1Ω
0.33A
r


1Ω
0.33A
r

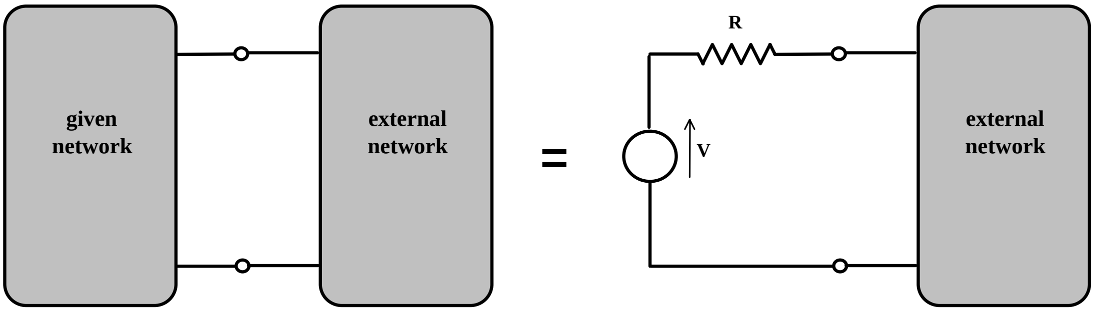
THEVENIN'S THEOREM

"So far as any external network connected between two terminals of a given network is concerned, the given network may be replaced by a voltage source, V, in series with a resistance, R

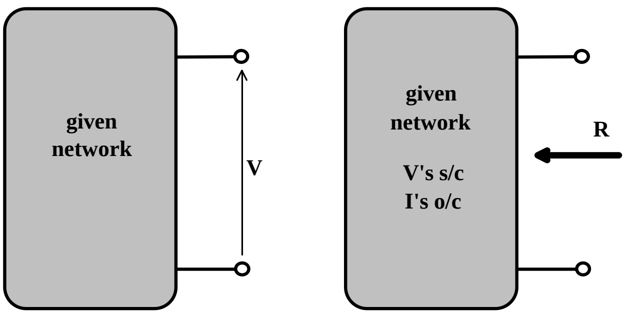
The magnitude of the voltage V is equal to the voltage appearing between the two terminals with the external network disconnected.

The resistance R is the resistance of the given network between the terminals with all sources reduced to zero, i.e. voltage sources short-circuit, current sources open-circuit."

Schematic:

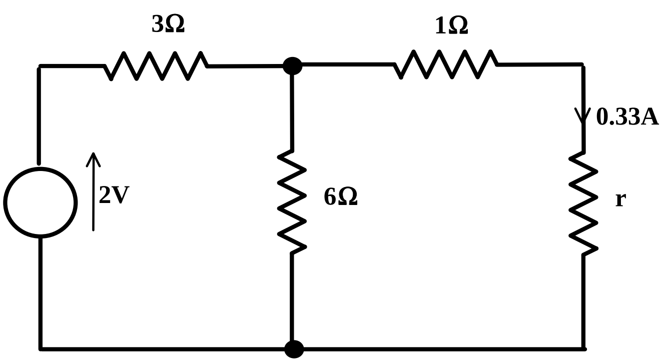


where:

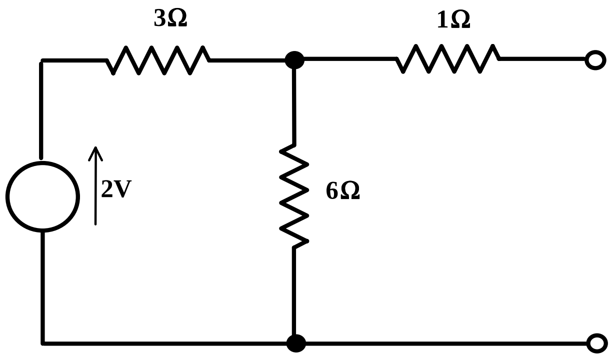


Example

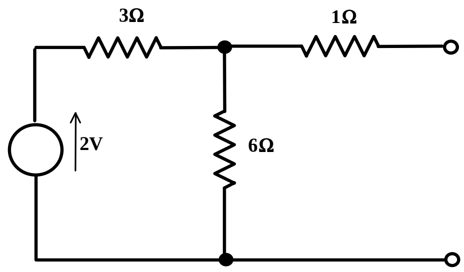
In the previous example, an alternative approach is to use Thevenin's Theorem, with the resistance r being the 'external network'.



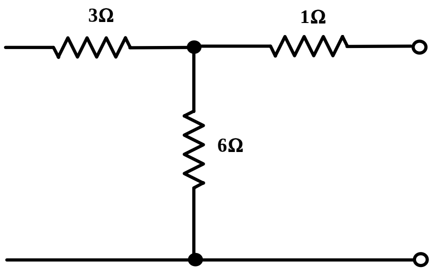
We need to find the Thevenin equivalent of the given network:



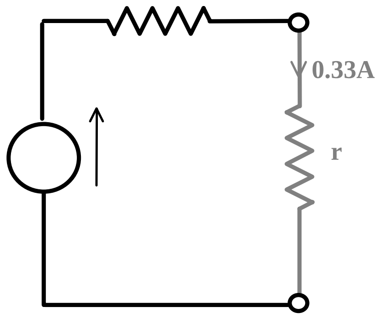
to find open-circuit voltage:



to find equivalent resistance:



so the Thevenin equivalent is:



An example of what Thevenin's Theorem can (and can't) do

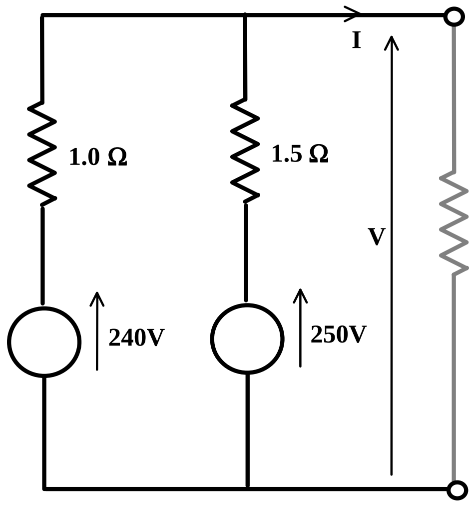
Two battery supplies, with the following parameters, are to operate in parallel:

supply	open-circuit voltage (V)	internal resistance (Ω)
A	240	1.0
B	250	1.5

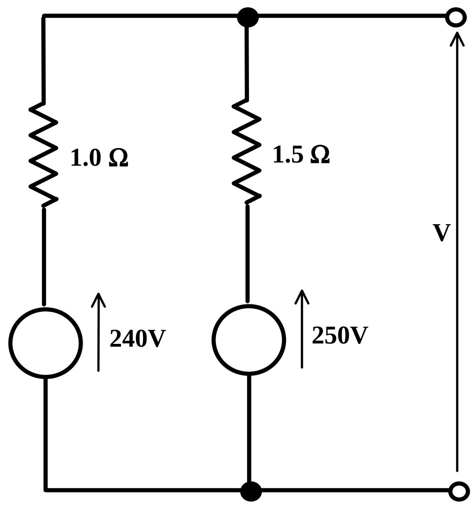
Calculate the terminal voltage of the combined supply when the load current is: i) 10A, ii) 20A. For a load current of 10A, calculate the current supplied by each battery.

Solution

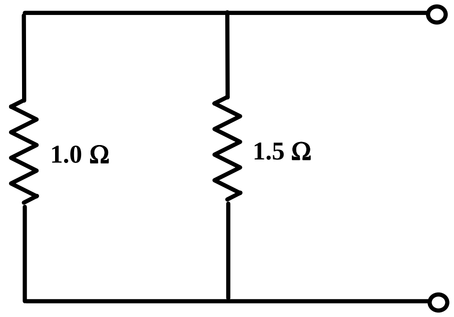
Equivalent circuit:



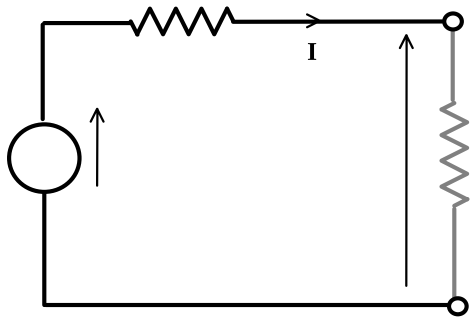
to calculate open-circuit voltage



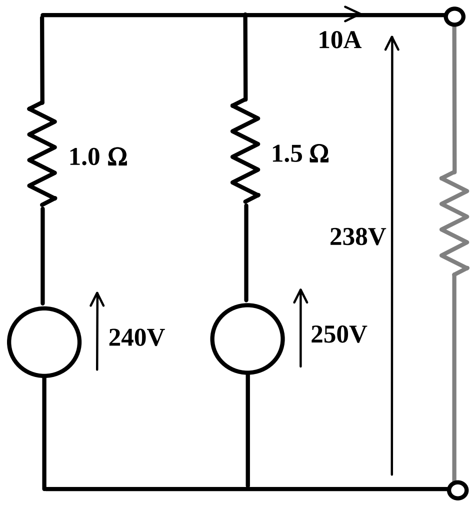
to calculate Thevenin resistance:



with Thevenin equivalent

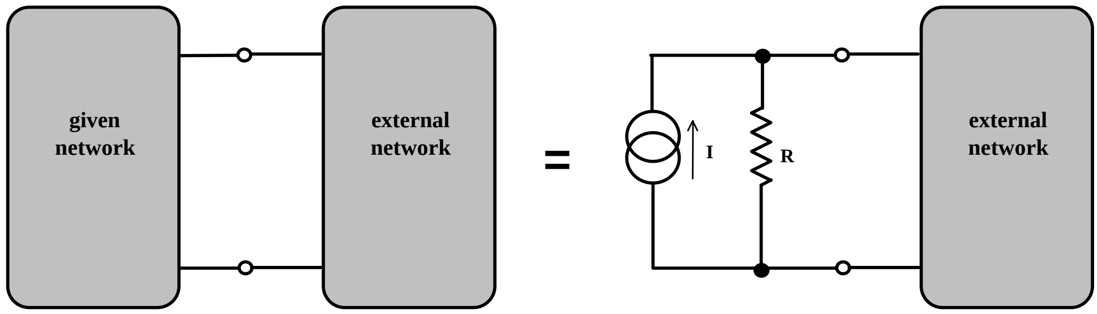


For a load current of 10A, to find the battery currents we must go back to the original circuit:

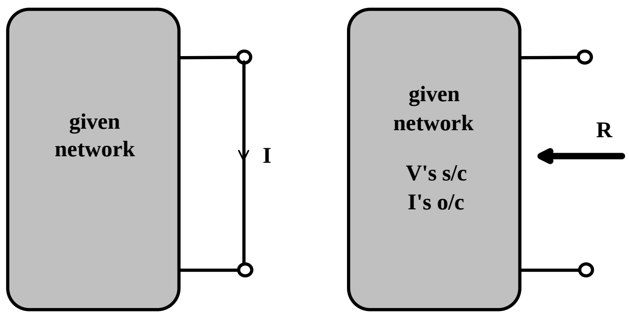


NORTON'S THEOREM

Because of source equivalence, any network can be reduced to its Norton Equivalent:



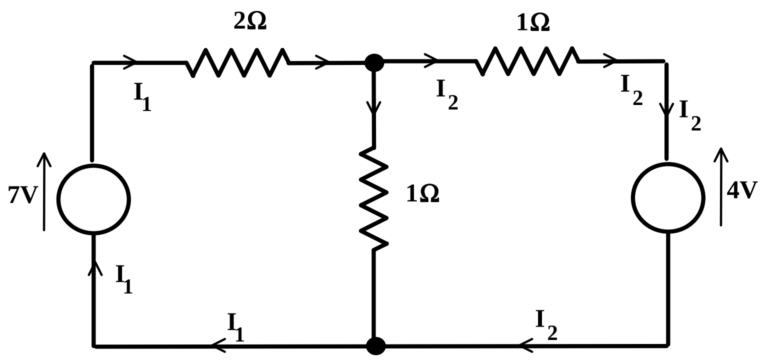
where:



TECHNIQUES OF CIRCUIT ANALYSIS

MESH CURRENT ANALYSIS

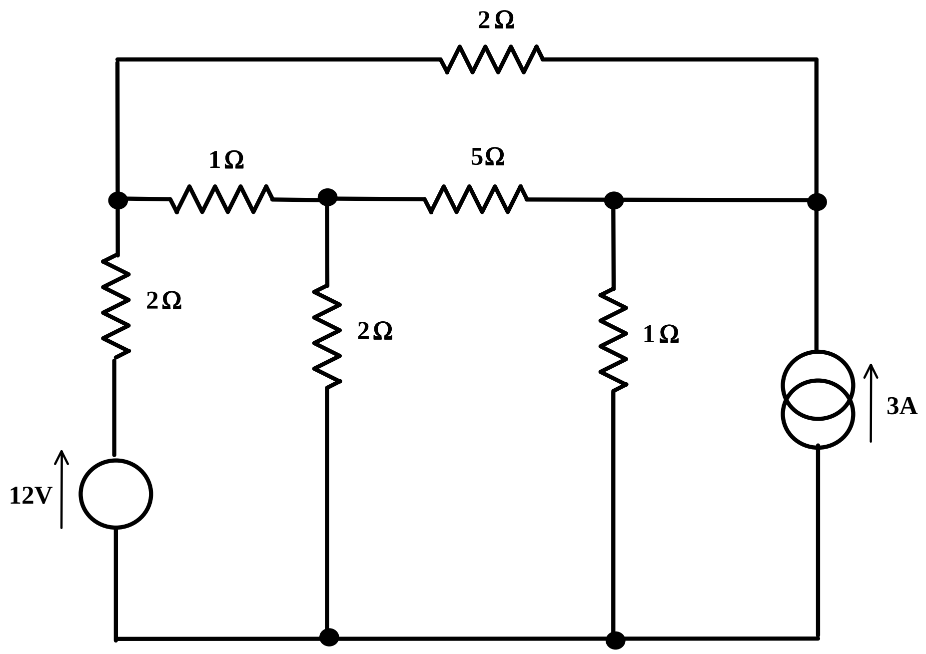
(also known as circulating current or loop current analysis)



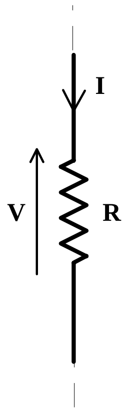
GENERAL PROCEDURE

1. Define mesh currents
2. Express branch currents in terms of loop currents, using Kirchoff's Current Law
3. Calculate the voltage across each element using Ohm's Law
4. Write down the mesh current equations using Kirchoff's Voltage Law.

EXAMPLE



POWER



In resistance, R, power dissipated,
$P = VI = I^2R = V^2/R$

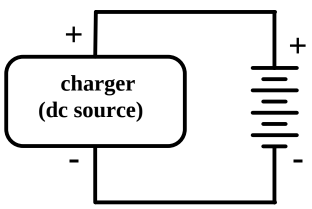
and in time t, energy dissipated

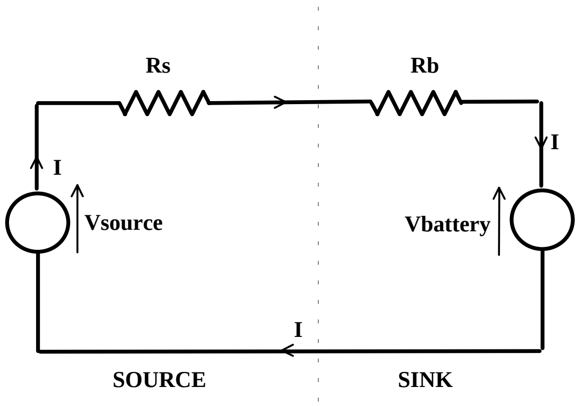
$$W = \int_0^t VI.dt = VIt$$

for constant V and I

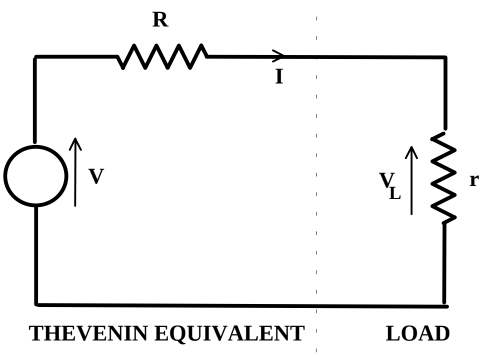
<u>SI Units</u>
units of power are Watts (W)
units of energy are Joules (J)

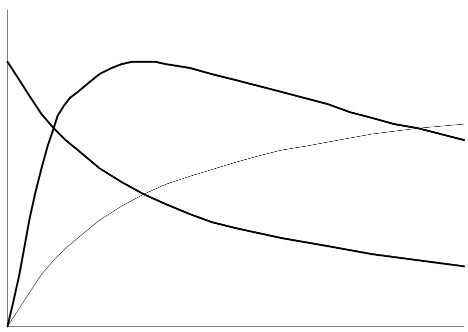
In a circuit the voltage and current sources may produce <u>or</u> absorb power. For example, consider a battery charger and its equivalent circuit:





POWER TRANSFER FROM A REAL SOURCE TO A LOAD





$$I = \frac{V}{R + r} \qquad V_L = rI = \frac{Vr}{R + r} \qquad P = V_L I = \frac{V^2 r}{(R + r)^2}$$

Given: V, R find r to maximise P.

$$\frac{dP}{dr} = \frac{\left[(R+r)^2 \frac{d}{dr}\left\{V^2 r\right\}\right] - \left[V^2 r \frac{d}{dr}\left\{(R+r)^2\right\}\right]}{(R+r)^4} = \frac{\left[(R+r)^2\left\{V^2\right\}\right] - \left[V^2 r.2.\left\{(R+r)\right\}\right]}{(R+r)^4} = 0$$

for maximum power. So:

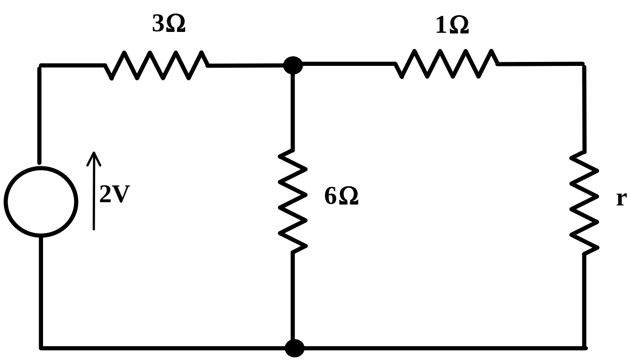
$$\left[(R+r)^2\left\{V^2\right\}\right] - \left[V^2 r.2.\left\{(R+r)\right\}\right] = 0$$

$\therefore \left[(R+r)\right] - \left[r.2\right] = 0$ or <u>r = R</u>

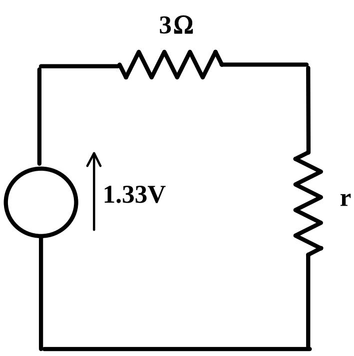
Maximum power = $P_{max} = \frac{V^2 r}{(R+r)^2}$ with r = R: $\underline{P_{max} = \frac{V^2}{4R}}$

Example

Find r to give maximum power



Thevenin equivalent of the circuit connected to r is:



so for maximum power in r, its value should be 3Ω.

The power dissipated is then: $P_{max} = 1.33^2/(4*3)$ W = 0.147 W

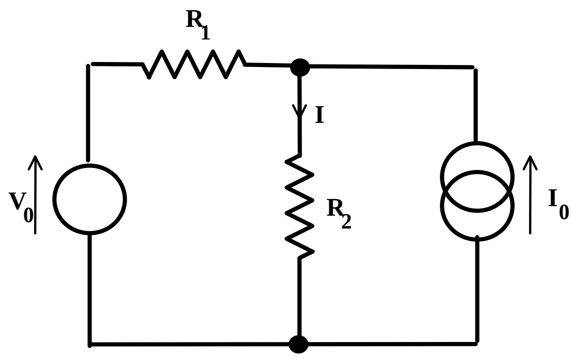
<u>Pause for thought:</u> How would you calculate the power delivered by the 2V source?

LINEARITY

An element or network is linear if its behaviour is independent of the magnitude of the voltage or current applied to it. A network is non-linear if one or more elements within it are non-linear. Network Theorems (Thevenin, Norton, Superposition) apply only to linear networks.

SUPERPOSITION

As an introduction to the Superposition Theorem, consider the following circuit:



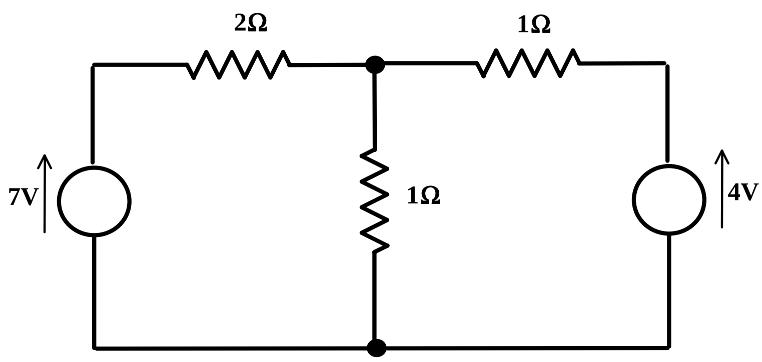
$$\frac{V_0 - R_2 I}{R_1} + I_0 - I = 0$$

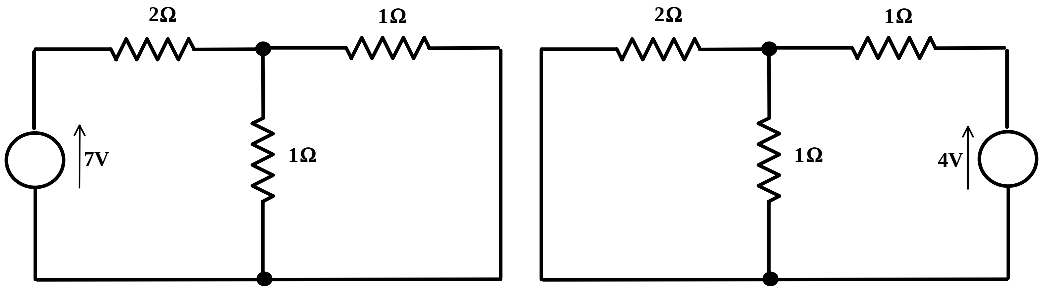
$$V_0 - R_2 I + R_1 I_0 - R_1 I = 0$$

$$I = \left\{ \frac{V_0}{R_1 + R_2} \right\} + \left\{ \frac{R_1 I_0}{R_1 + R_2} \right\}$$

Superposition Theorem
"The total current flowing in any branch of a network is the algebraic sum of the currents which would flow in that branch if each source was acting alone, with the other sources reduced to zero"

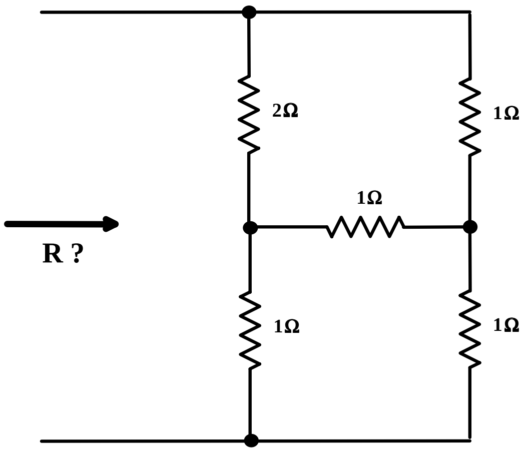
Example





CIRCUIT SIMPLIFICATION: 3-TERMINAL SYSTEMS

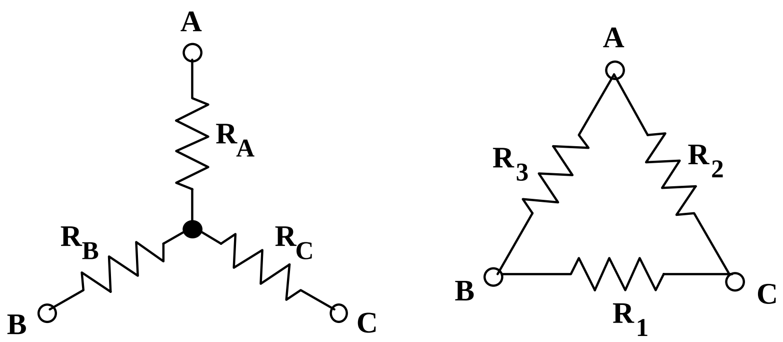
Using the basic techniques of combining resistances in series and parallel, it is not possible to simplify the network of resistances shown below into a single equivalent resistance:



Star/delta and delta/star transformations

These transformations allow us to deal with problems like that shown above, by transforming one set of three resistances, connected between three terminals of a network, into an equivalent set of resistances connected between the same terminals and exhibiting identical electrical behaviour.

Star connection Delta connection



The alternative connections - star and delta - of the sets of three resistances are shown in the diagram. The transformations allow us to replace one set of resistances, e.g. R_1,

R_2, R_3 in the delta connection, by an equivalent set of three resistances with the opposite connection, e.g. R_A, R_B, R_C in the star connection. Algebraic relationships between the two sets of resistances can be derived by remembering that the two connections are exactly equivalent.

For example, the **delta/star transformation** is obtained by considering the effective resistance between pairs of terminals, with the other terminal open-circuit.

terminal pair	star	delta
A - B	$R_A + R_B$ =	$R_3 \| (R_1 + R_2) = \frac{R_3(R_1+R_2)}{R_1+R_2+R_3} = \frac{R_1.R_3 + R_2.R_3}{R_1+R_2+R_3}$ (1)
B - C	$R_B + R_C$ =	$R_1 \| (R_2 + R_3) = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} = \frac{R_1.R_2 + R_1.R_3}{R_1+R_2+R_3}$ (2)
C - A	$R_C + R_A$ =	$R_2 \| (R_1 + R_3) = \frac{R_2(R_1+R_3)}{R_1+R_2+R_3} = \frac{R_1.R_2 + R_2.R_3}{R_1+R_2+R_3}$ (3)

Subtract (2) from (3):

$$R_A - R_B = \frac{R_2.R_3 - R_1.R_3}{R_1+R_2+R_3} \quad (4)$$

then take {(1) + (4)}/2:

$$R_A = \frac{R_2.R_3}{R_1+R_2+R_3}$$ and similarly: $$R_B = \frac{R_1.R_3}{R_1+R_2+R_3} \qquad R_C = \frac{R_1.R_2}{R_1+R_2+R_3}$$

The **star/delta transformation** can be found using a similar approach:

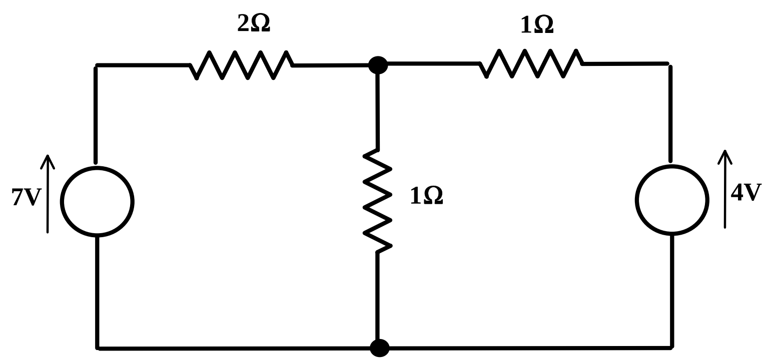
$$R_1 = \frac{R_A.R_B + R_B.R_C + R_C.R_A}{R_A}$$

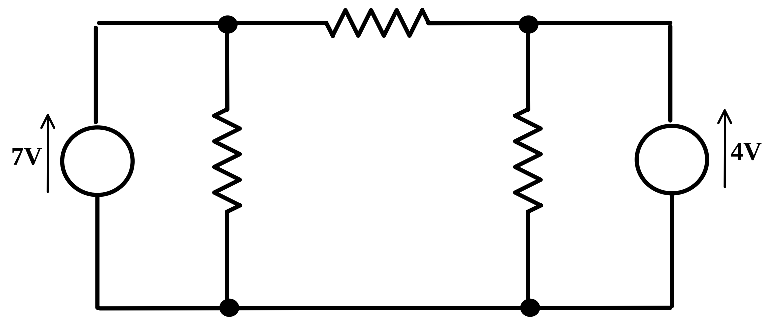
$$R_2 = \frac{R_A.R_B + R_B.R_C + R_C.R_A}{R_B}$$

$$R_3 = \frac{R_A.R_B + R_B.R_C + R_C.R_A}{R_C}$$

Example

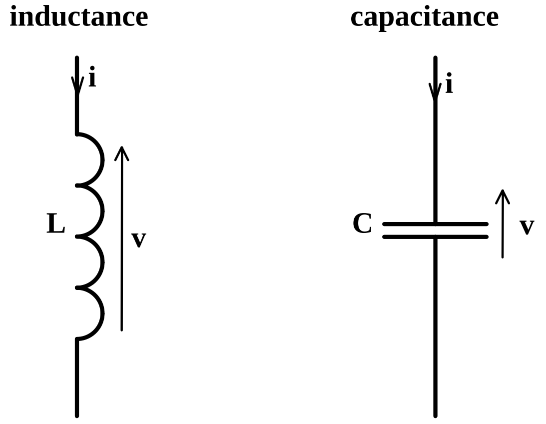
Use the star/delta transformation to solve:





TRANSIENTS IN ELECTRICAL CIRCUITS

Effect of switching, involves two further circuit elements:



$$v = L\frac{di}{dt} \qquad\qquad i = C\frac{dv}{dt}$$

Units:	L: Henrys (H)	C: Farads (F)

<u>Note on Units</u>

Units of capacitance are inconveniently large, so usually deal in µF, nF or pF:

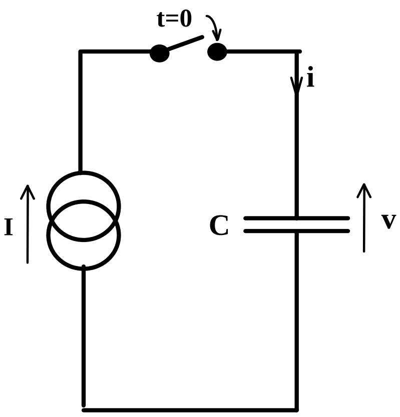
T	$x10^{12}$	Terra	e.g. TWh
G	$x10^{9}$	Giga	e.g. GHz
M	$x10^{6}$	Mega	e.g. MΩ
k	$x10^{3}$	kilo	e.g. kV
-	$x10^{0}$		
m	$x10^{-3}$	milli	e.g. mW
µ	$x10^{-6}$	micro	e.g. µH
n	$x10^{-9}$	nano	e.g. ns
p	$x10^{-12}$	pico	e.g. pF

<u>Analysis of time-varying voltages and currents</u>

Two categories to consider: transients - effect of switching dc
ac -effect of sine waves in steady-state

Kirchoff's Laws, Techniques of Circuit Analysis and Network Theorems apply in circuits with time-varying voltages and currents.

TRANSIENT ANALYSIS

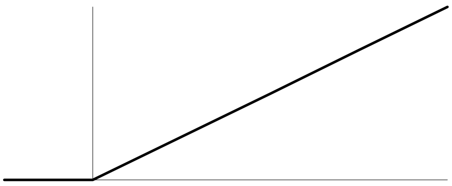


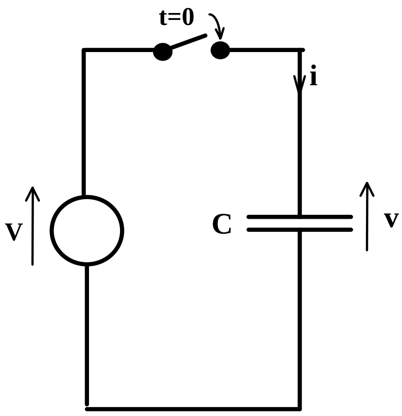
For t>0

$$i = I = C\frac{dv}{dt}$$

$$\therefore \frac{dv}{dt} = \frac{I}{C}$$

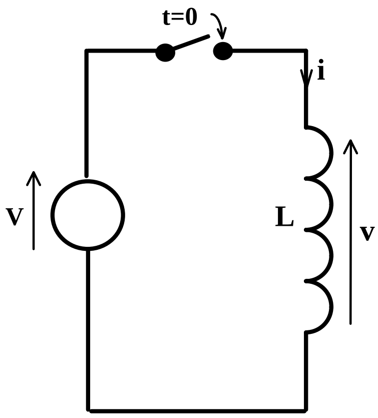
$$\therefore v = \frac{I}{C}t$$





At t=0, v: 0→V

$$i = C\frac{dv}{dt} \rightarrow \infty$$

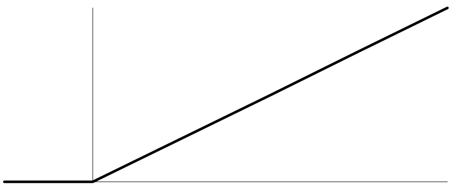


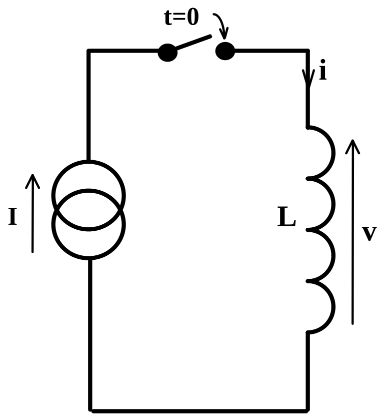
For t>0

$$v = V = L\frac{di}{dt}$$

$$\therefore \frac{di}{dt} = \frac{V}{L}$$

$$\therefore i = \frac{V}{L}t$$

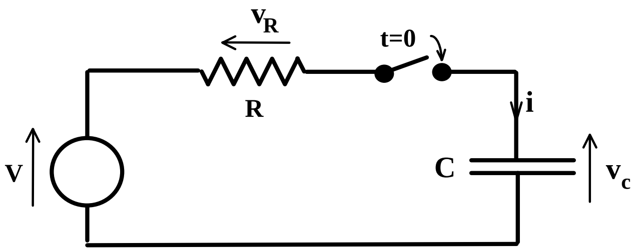




At t=0 i: 0→I

$$v = L\frac{di}{dt} \rightarrow \infty$$

R-C in series

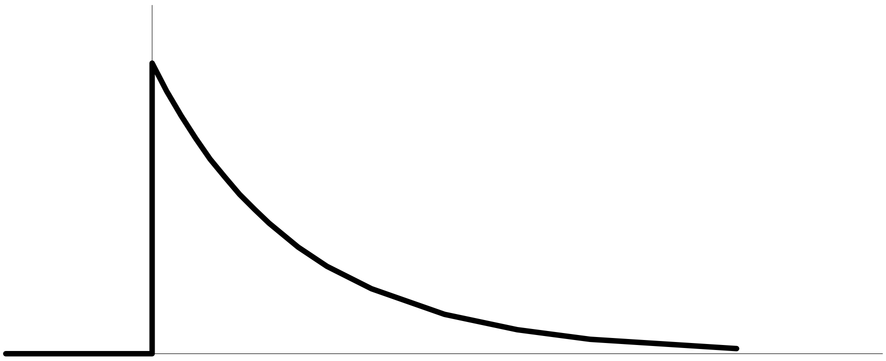


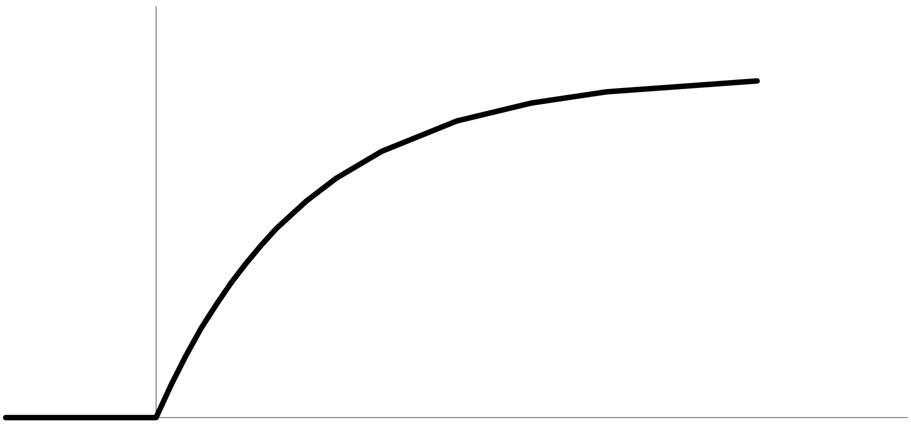
With the switch closed for t≥0, Kirchoff's Voltage Law can be applied:

$$v_R + v_c - V = 0 \qquad \text{but} \qquad v_R = R.i$$

so:

$$i = \frac{V - v_c}{R} \qquad \text{and} \qquad v_c = \frac{1}{C}\int i.dt$$



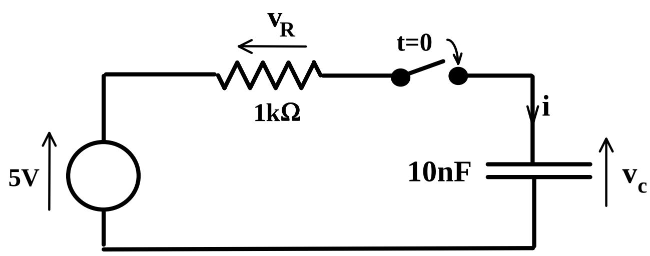


$$i = \frac{V}{R}.e^{\left(-\frac{t}{RC}\right)} = \frac{V}{R}.e^{\left(-\frac{t}{T}\right)} \qquad v_c = V\left[1 - e^{\left(-\frac{t}{RC}\right)}\right] = V\left[1 - e^{\left(-\frac{t}{T}\right)}\right]$$

where time constant T = R.C

A *large* time constant gives a *slow* response
A *small* time constant gives a *fast* response

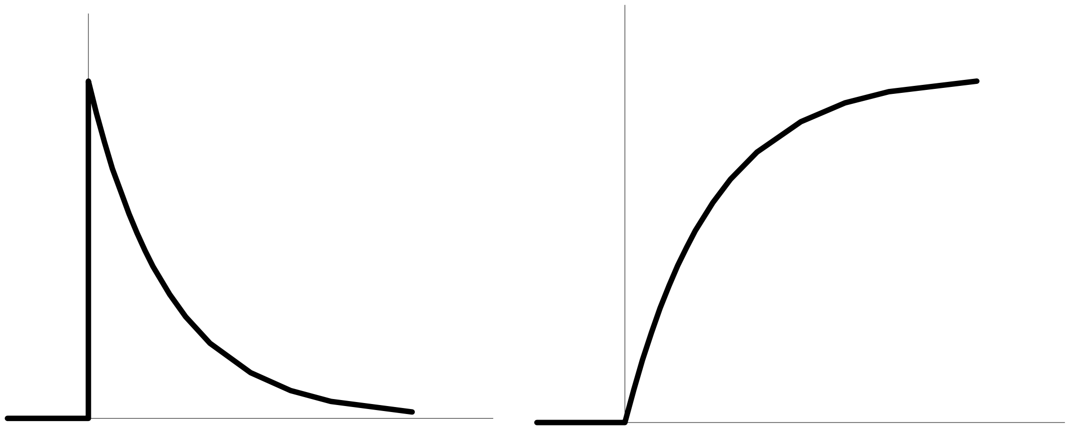
Example



Time constant, RC = $10^3 x 10 x 10^{-9}$ s = 10μs

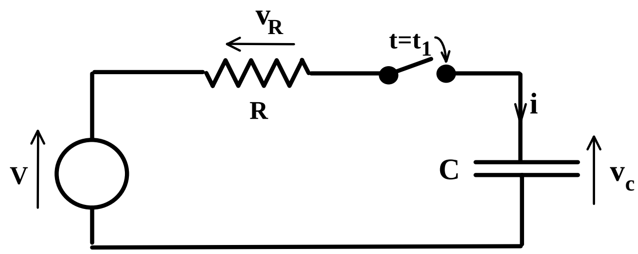
Final value of capacitor voltage = 5V

Initial value of current = V/R = 5/(10^3) = 5mA



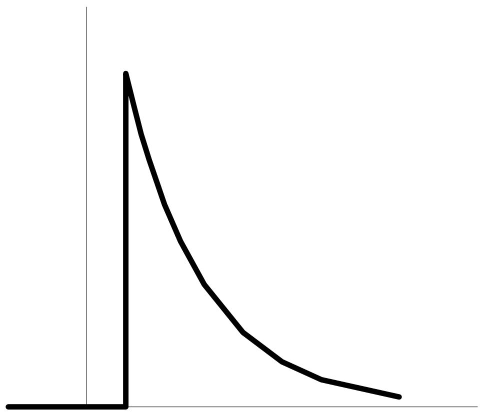
Different initial conditions: different switching time

Suppose the switch closes at a different time, e.g. $t=t_1$?



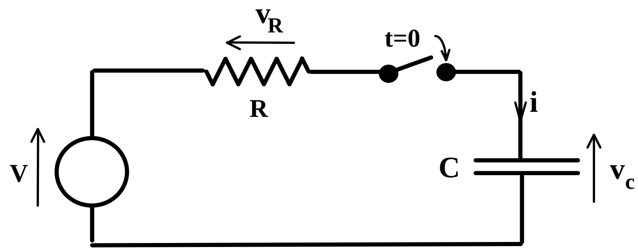
$$i = \frac{V}{R}.e^{\left(-\frac{t-t_1}{RC}\right)} = \frac{V}{R}.e^{\left(-\frac{t-t_1}{T}\right)}$$

where time constant T = R.C (as before) and the solution applies for $t \geq t_1$.



Different initial conditions: initial capacitor voltage

Suppose the switch closes at time t=0, with the capacitor already charged to a voltage $v_c = V_0$?



The limits of integration are different, because of the initial charge on the capacitor. At the moment the switch closes Kirchoff's Voltage Law must be satisfied, so:

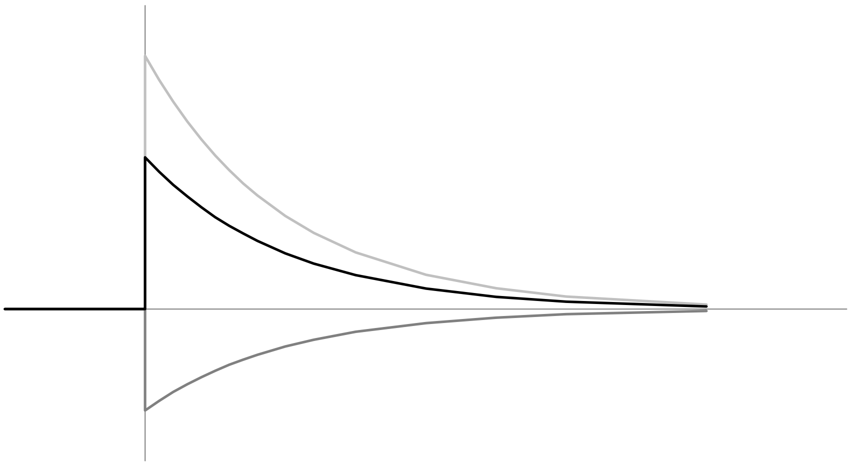
$$v_R + v_c - V = 0$$

and substituting:

$$R.i \;\; + V_0 - V = 0$$

so:

$$\text{initial current (t=0) } i \;\; = \left(V - V_0\right) / R$$

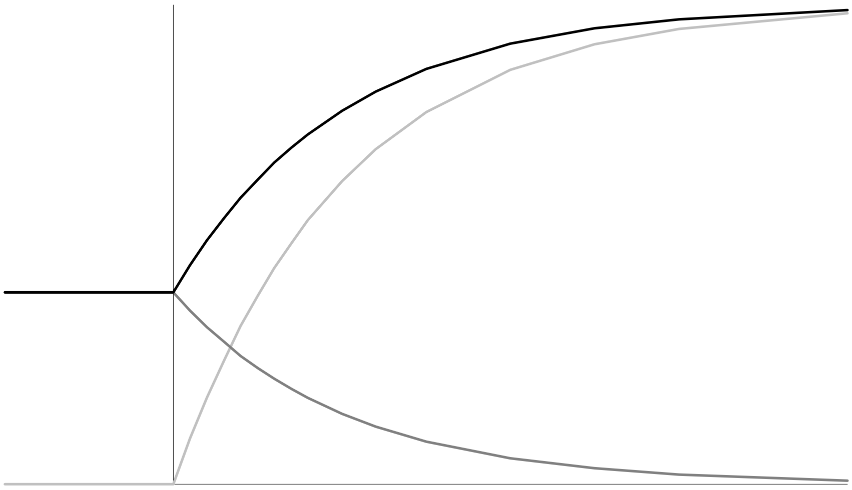


$$i = \frac{\left(V - V_0\right)}{R}.e^{\left(-\frac{t}{RC}\right)} = \frac{\left(V - V_0\right)}{R}.e^{\left(-\frac{t}{T}\right)} = \frac{V}{R}.e^{\left(-\frac{t}{T}\right)} + \frac{\left(-V_0\right)}{R}.e^{\left(-\frac{t}{T}\right)}$$

where time constant T = R.C (as before) and the solution applies for $t \geq 0$. Note that the solution consists of two components: an illustration of:

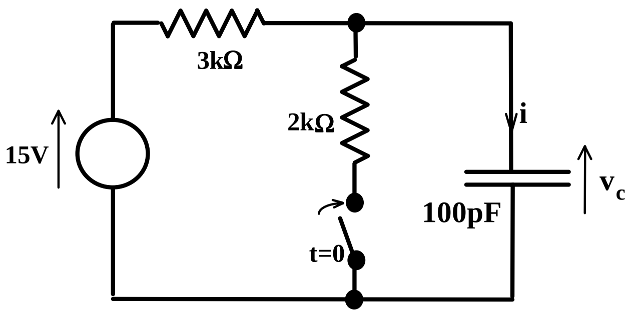
The solution for capacitor voltage as a function of time may be inferred using the same principle:

$$v_C = V\left[1 - e^{\left(-\frac{t}{T}\right)}\right] + V_0 e^{\left(-\frac{t}{T}\right)}$$



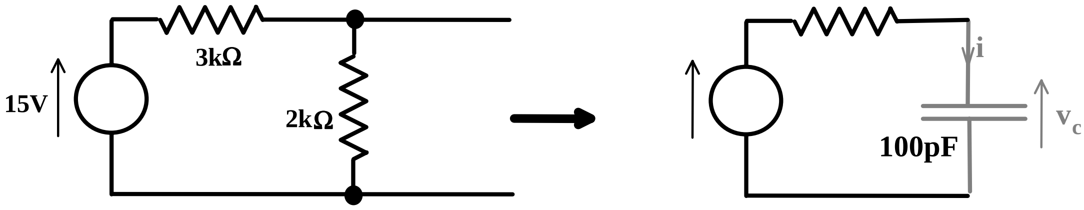
More complex circuits

Circuits involving more complex arrangements of sources are solved by finding the Thevenin equivalent of the circuit ‘seen’ by the energy storage element (L or C), as in the following **example**:



The first step in finding the capacitor voltage v_c, as a function of time, is to establish the *initial conditions,* i.e. the values of all electrical quantities before the switch closes. Assuming the circuit has been constructed ‘a long time’ ( ) previously, the initial value of v_c is:

Next, find the Thevenin equivalent of the circuit connected to the capacitor when the switch is closed:

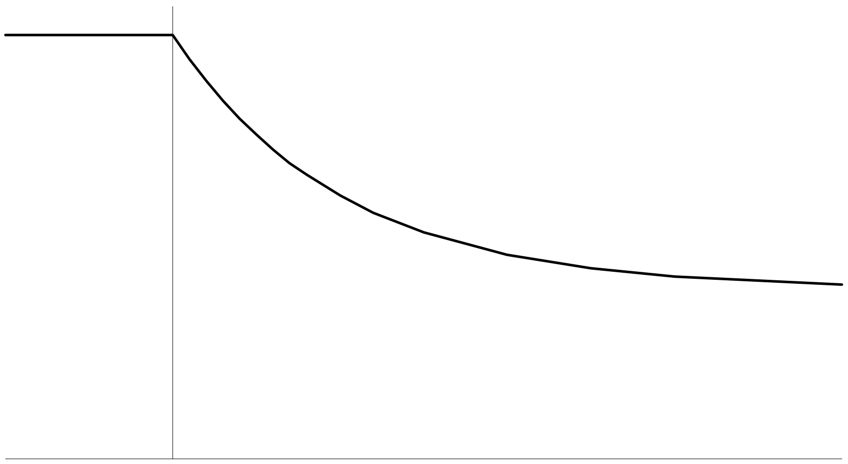


So the time constant for the change in capacitor voltage = R.C =

Steady-state value of capacitor voltage =

and, if necessary, an algebraic expression for the variation of capacitor voltage with time can be written:

v_c =

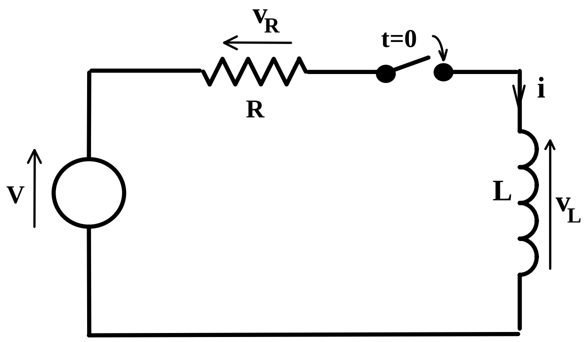


The variation of capacitor current with time can be derived using the relation:

$$i = C\frac{dv_C}{dt}$$

or by using superposition:

R-L in series

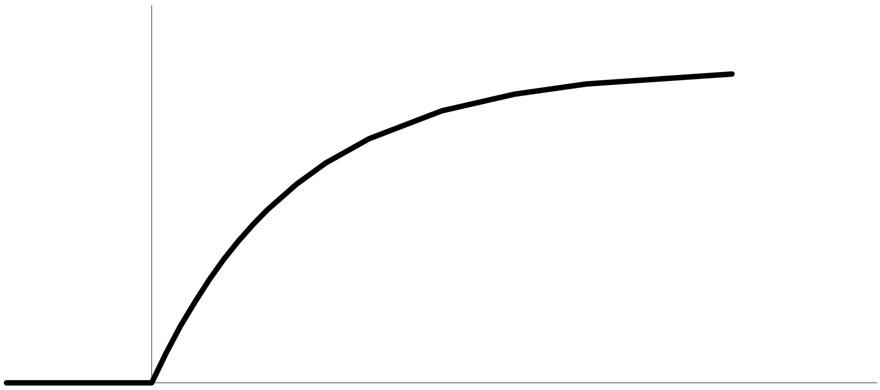


With the switch closed for t≥0, Kirchoff's Voltage Law can be applied:

$$v_R + v_L - V = 0$$

and substituting for the voltage-current relations for R and L:

$$Ri + L\frac{di}{dt} = V \qquad \text{so:} \quad \frac{di}{dt} = \frac{V - Ri}{L}$$

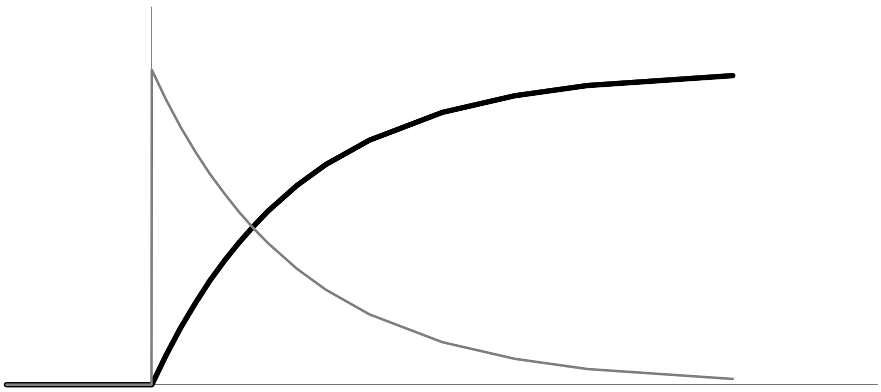


$$i = \frac{V}{R}\left[1 - e^{-Rt/L}\right] = \frac{V}{R}\left[1 - e^{-t/T}\right]$$

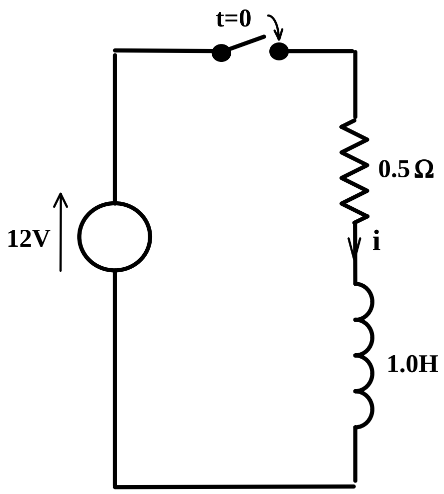
where time constant, T = L/R

$$v_R = Ri = V\left[1 - e^{-Rt/L}\right] = V\left[1 - e^{-t/T}\right]$$

$$v_L = V - v_R = V\left[e^{-Rt/L}\right] = V\left[e^{-t/T}\right]$$



Example

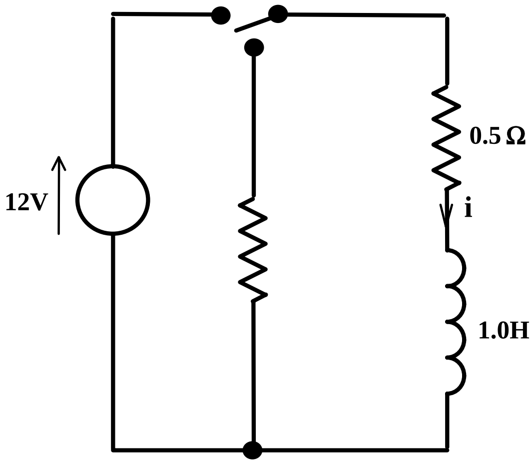


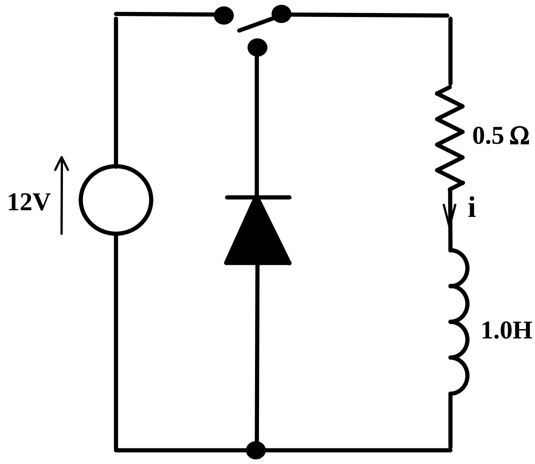
Time constant, T =

Steady-state current =

but what happens when the switch is opened?

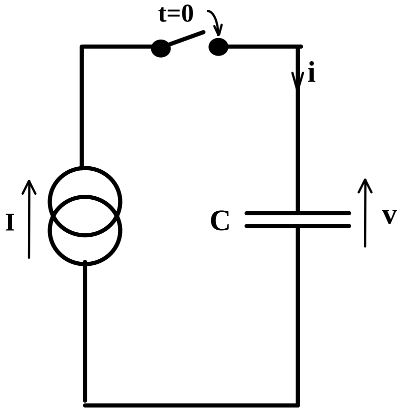
Alternative circuits:





ENERGY STORAGE

CAPACITANCE



For t>0

$$i = I = C\frac{dv}{dt}$$

$$\therefore \frac{dv}{dt} = \frac{I}{C}$$

$$\therefore v = \frac{I}{C}t$$

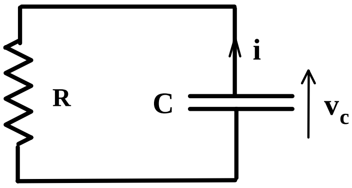
Power = vI = I^2t/C

Energy input in time t = $\int_0^t \frac{I^2 t}{C}.dt = \frac{1}{2}.\frac{(It)^2}{C}$

but at time t, the capacitor voltage V=It/C, so:

<u>Energy input to the capacitor = ½CV^2</u>

This energy is stored in the capacitor and can be released back into the electrical circuit. For example if the current source is replaced by a resistance R, the capacitor discharges from a voltage V to zero with a time constant RC:



$$i = \frac{V}{R}.e^{-t/RC}$$

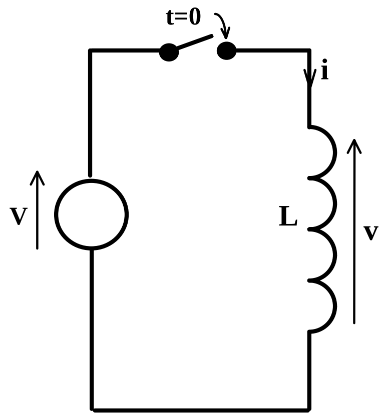
Power dissipated in resistance, R = $= i^2 R = \left(\frac{V}{R}.e^{-t/RC}\right)^2 R = \frac{V^2}{R}.e^{-2t/RC}$

So the total energy dissipated in R as the capacitor discharges

$$= \int_0^\infty i^2 R.dt = \frac{V^2}{R}\int_0^\infty e^{-2t/RC}.dt = \frac{V^2}{R}\left[-\frac{RC}{2}.e^{-2t/RC}\right]_0^\infty = -\frac{V^2 C}{2}[0-1] = \tfrac{1}{2}CV^2$$

Hence the energy originally input to the capacitor (½CV^2) has been **stored** and is then available to be transferred to the resistance.

INDUCTANCE



For t>0

$$v = V = L\frac{di}{dt}$$
$$\therefore \frac{di}{dt} = \frac{V}{L}$$
$$\therefore i = \frac{V}{L}t$$

Power = Vi = V^2t/L

Energy input in time t = $\int_0^t \frac{V^2 t}{L}.dt = \frac{1}{2}.\frac{(Vt)^2}{L}$

but at time t, the inductor current I=Vt/L, so:

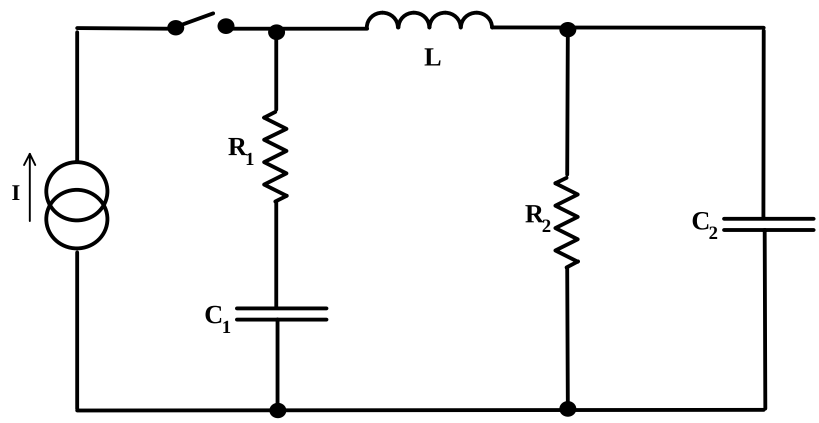
Energy input to the inductor = ½LI^2

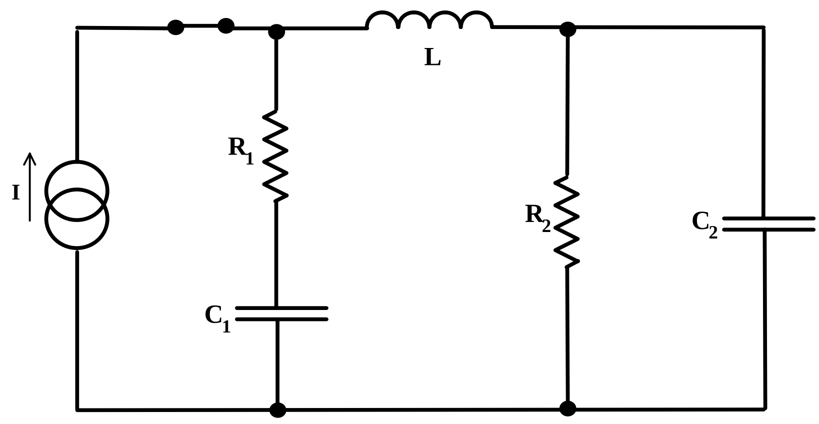
SUMMARY OF ENERGY STORAGE ELEMENTS

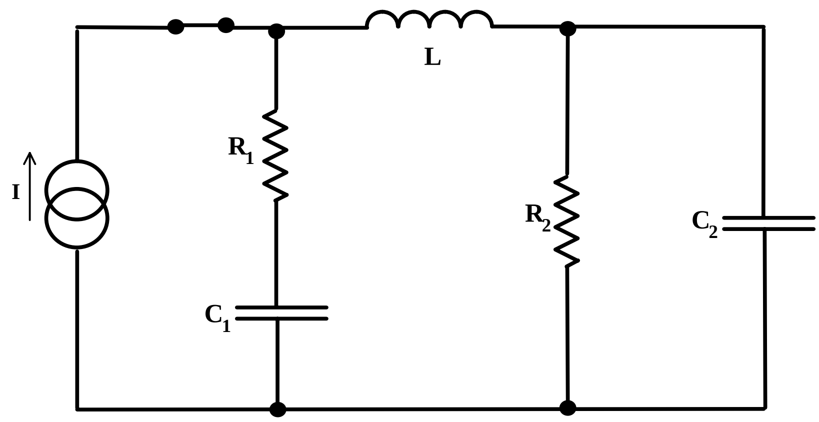
CAPACITANCE	**INDUCTANCE**
$i = C\frac{dv}{dt}$	$v = L\frac{di}{dt}$
Capacitor voltage cannot change instantaneously	Inductor current cannot change instantaneously
Time constant = RC	Time constant = L/R
In the steady-state (after many time constants), capacitor current =0	In the steady-state (after many time constants), inductor voltage =0
Energy storage = ½CV^2	Energy storage = ½LI^2

HIGHER-ORDER SYSTEMS

Circuits involving more than one energy storage element lead to solutions with higher-order differential equations. The solution of these circuits is most easily undertaken using Laplace Transformations (see later lecture courses on Linear Systems), but sometimes it is sufficient to deduce the initial and steady-state conditions in the circuit, as in the following example:




L
R1
I
R2
C2
C1


L
R1
I
R2
C2
C1

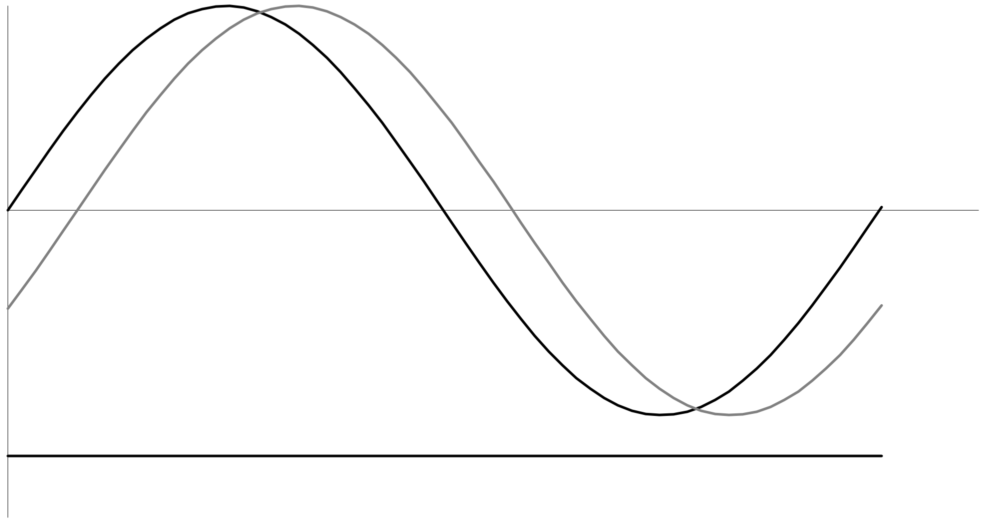
AC CIRCUITS

AC = alternating current

Consider the behaviour of circuits when voltage and current sources are sinusoidal functions of time. Steady-state: a 'long time' after the source(s) were connected.

Why sinusoidal? Integral and differential with respect to time is also sinusoidal. All other repetitive waveforms can be expressed as the sum of sinusoidal waveforms (Fourier Series)

<u>Parameters of sine waves</u>



$v_1 = V.\sin(\omega t)$

$\omega =$

$f =$

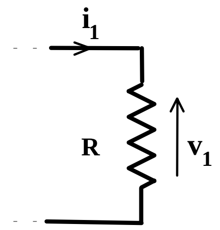
$V =$

$2V =$

$v_2 = V.\sin(\omega t - \theta)$

$\theta =$

Power



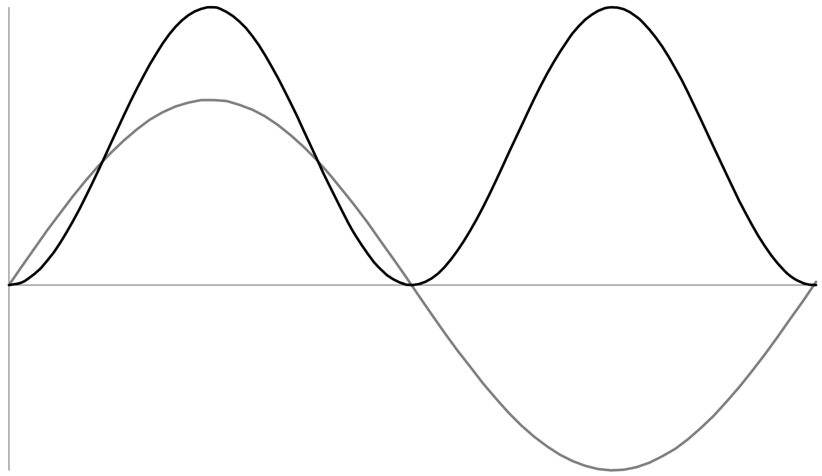
$v_1 = V.\sin(\omega t)$

From Ohm's Law: $i_1 = \frac{v_1}{R} = \frac{V}{R}.\sin(\omega t)$

so in this linear circuit element current and voltage have the same ...

Power dissipated in resistance R?

$$P = v_1.i_1 = V.\sin(\omega t).\frac{V}{R}.\sin(\omega t) = \frac{V^2}{R}.\sin^2(\omega t) = \frac{V^2}{R}.\frac{1}{2}.[1-\cos(2\omega t)]$$



The *instantaneous* power varies with time at a frequency of:

$\text{Average} = \frac{V^2}{2R} = \frac{\left(V/\sqrt{2}\right)^2}{R}$, so the voltage V.sin(ωt) applied to the resistor R produces a mean power dissipation equal to that produced by a dc source of $\left(V/\sqrt{2}\right)$.

$\left(V/\sqrt{2}\right)$ is the root mean square (r.m.s.) value of the sinusoidal voltage.

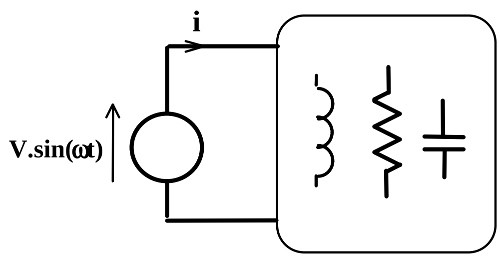
For a sinusoidal voltage, V.sin(ωt) :

peak = V
peak-to-peak = 2V
root-mean-square = $\left(V / \sqrt{2}\right)$

Unless stated otherwise, it is the rms value which is quoted when referring to ac quantities.

Steady-state ac circuit analysis

Introduction
Here is a simple circuit in which a voltage source {v = V.sin(ωt)} is supplying a collection of resistors, capacitors and inductors:



We might assume, correctly, that the current flowing from the source has the same frequency, but we do not know its magnitude or phase angle relative to the voltage. In general we could write:

$$i = I.\sin(\omega t + \theta)$$

and the analysis problem becomes one of finding the current magnitude, I, and phase angle, θ. It is possible to perform basic circuit analysis using these sinusoidal time functions of voltage and current, but you need to be very good at dealing with trigonometric functions and there is an easier way:

'j' notation

We have seen that a voltage V.sin(ωt) applied to a circuit produces a current I.sin(ωt+θ). Suppose instead the voltage applied had been v=V.cos(ωt). The current magnitude and phase angle would be unchanged, so i= I.cos(ωt+θ), so:

$$V.\sin(\omega t) \quad \rightarrow \quad I.\sin(\omega t+\theta)$$

$$V.\cos(\omega t) \quad \rightarrow \quad I.\cos(\omega t+\theta)$$

These two results can be combined, and the analysis considerably simplified, if we consider the voltage applied to the circuit:

$$v = V.\cos(\omega t) + jV.\sin(\omega t) = V.\{\cos(\omega t) + j\sin(\omega t)\}$$

where $j=\sqrt{-1}$

Because in general $e^{jx} = \cos(x) + j\sin(x)$:

$$v = V.e^{j\omega t}$$

Using the Superposition Theorem, the applied voltage with its two components must cause a current with two corresponding components to flow around the circuit:

$$i = I.\cos(\omega t + \theta) + jI.\sin(\omega t + \theta) = I.\{\cos(\omega t + \theta) + j\sin(\omega t + \theta)\}$$

which can be written:

$$i = I.e^{j(\omega t + \theta)} = I.e^{j(\omega t)}.e^{j(\theta)} = \tilde{I}.e^{j\omega t}$$

where:

$$\tilde{I} = I.e^{j(\theta)}$$

is a complex number which has a magnitude (modulus), I, which is equal to the magnitude of the current flowing around the circuit, and an angle (argument), θ, which is equal to the phase angle of the current relative to the voltage. $\tilde{I}$ is usually referred to as the *phasor* current.

The voltage v can be expressed as a phasor quantity also:

$$v = V.e^{j(\omega t)}.e^{j(0)} = \tilde{V}.e^{j\omega t}$$

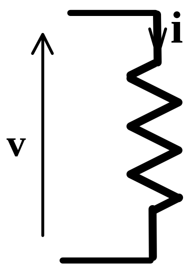
where:

$$\tilde{V} = V.e^{j0}$$

The angle associated with the phasor quantity $\tilde{V}$ is zero. In this problem $\tilde{V}$ is the *reference* phasor and the phase angles of all the other phasor quantities are defined relative to this reference.

Relationships between phasor voltages and currents in R, C, L

Complex impedance of resistance



$v = Ri$

if: $i = \tilde{I}e^{j\omega t}$ ..... $v = \tilde{V}e^{j\omega t}$
so:

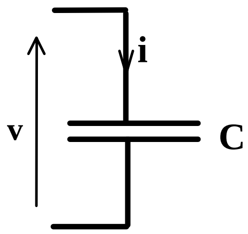
$\tilde{V}e^{j\omega t} = R\tilde{I}e^{j\omega t}$

$\therefore \tilde{V} = R\tilde{I}$

$\therefore \dfrac{\tilde{V}}{\tilde{I}} = R$

the ratio of the phasor voltage to the phasor current is called the *complex impedance* of the circuit or circuit element. The magnitude of the complex impedance is equal to the ratio of voltage to current magnitudes and the angle of the complex impedance is equal to the phase angle of the voltage relative to the current. In this case we see that the complex impedance is simply the scalar quantity R, indicating that the voltage and current are in phase (relative phase angle is zero).

Complex impedance of capacitance



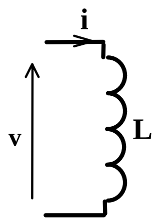
$$i = C\frac{dv}{dt}$$
$$i = \tilde{I}e^{j\omega t} \ldots\ldots v = \tilde{V}e^{j\omega t}$$
so:
$$\tilde{I}e^{j\omega t} = j\omega C\tilde{V}e^{j\omega t}$$

$$\therefore \tilde{V} = \frac{\tilde{I}}{j\omega C} = \frac{-j}{\omega C}\tilde{I} = \frac{1}{\omega C}.e^{j(-90^o)}\tilde{I}$$

$$\therefore \frac{\tilde{V}}{\tilde{I}} = \frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{1}{\omega C}.e^{j(-90^o)}$$

The complex impedance of the capacitor is 1/jωC, indicating that the voltage *lags* current by 90^o and capacitive reactance = 1/ωC

Complex impedance of inductance



$$v = L\frac{di}{dt}$$
$$i = \tilde{I}e^{j\omega t} \ldots\ldots v = \tilde{V}e^{j\omega t}$$
so:
$$\tilde{V}e^{j\omega t} = j\omega L\tilde{I}e^{j\omega t}$$

$$\therefore \tilde{V} = j\omega L\tilde{I} = \omega L.e^{j(90^o)}\tilde{I}$$

$$\therefore \frac{\tilde{V}}{\tilde{I}} = j\omega L = \omega L.e^{j(90^o)}$$

The complex impedance of the inductor is jωL, indicating that the voltage *leads* current by 90^o and inductive reactance = ωL

Definitions

if complex impedance Z = R + jX

R = resistance = Real{Z}
X = reactance = Imag.{Z}
[units of Z, R and X are Ohms]

and:
complex admittance Y = (1/Z) = G + jB

G = conductance = Real{Y}
B = susceptance = Imag.{Y}
[units of Y, G and B are Siemens (S)]

Note that in general: G ≠ 1/R and B ≠ 1/X

Notation

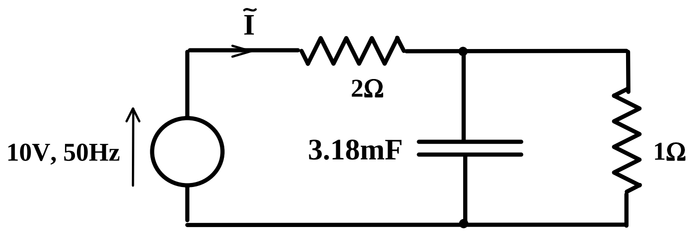
Writing $\tilde{V} = Ve^{j\theta}$ is inconvenient, so it is often abbreviated: $\tilde{V} = V\angle\theta$

Application of phasors and complex impedance

Complex impedances are subject to the same rules for circuit simplification as resistances in dc circuits: rules for combining in series or parallel are applicable to complex impedances, as are star/delta and delta/star transformations. The circuit theorems (Thevenin, Norton, Superposition) and techniques of circuit analysis (Nodal Voltage and Mesh Current) can be extended to phasor quantities and complex impedances.

Example

Calculate the phasor current $\tilde{I}$ in the following circuit:

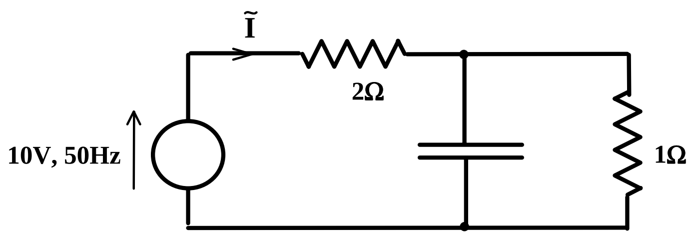


Convert capacitances and inductances into reactances

C = 3.18mF

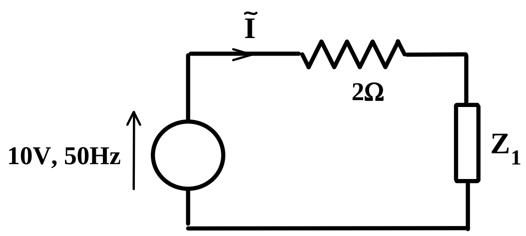
so capacitive reactance =

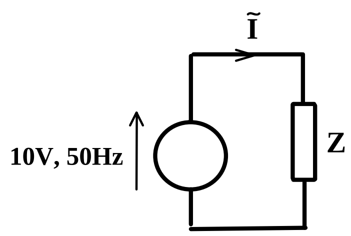
Hence the circuit becomes:



and combining the parallel circuit elements into a single complex impedance Z_1:

Z_1 =





so the total impedance, $Z = 2 + Z_1 =$

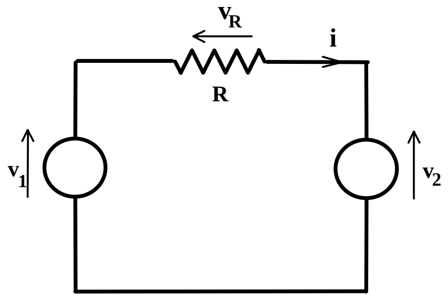
Therefore the phasor current is:

$$\tilde{I} = \frac{\tilde{V}}{Z} = \frac{10\angle 0^o}{2.54\angle -11.3^o} = 3.9\angle +11.3^o$$

Therefore the current magnitude is 3.9A (r.m.s.) and the current leads the voltage by a phase angle of 11.3^o.

Problems with several sources

General principle is to choose one source as the reference phasor and to express the phase angles of the other sources relative to the reference, as in the following example:



Given $v_1 = 5.\cos(\omega t)$ and $v_2 = 5.\cos(\omega t - 120^o)$, calculate the current, i.

Using phasors:

choose v_1 as the reference, so: $\tilde{V}_1 = 5\angle 0\ \text{V}$

then: $\tilde{V}_2 = 5\angle -120^o\ \text{V}$

from Kirchoff's Voltage Law: $\tilde{V}_R + \tilde{V}_2 - \tilde{V}_1 = 0$

so:

$$\tilde{V}_R = \tilde{V}_1 - \tilde{V}_2$$

$$\therefore \tilde{I} = \frac{\tilde{V}_1 - \tilde{V}_2}{1} = \frac{5\angle 0 - 5\angle -120}{1} = \frac{(5 + j0) - (5.\cos(-120) + j.5.\sin(-120))}{1}$$

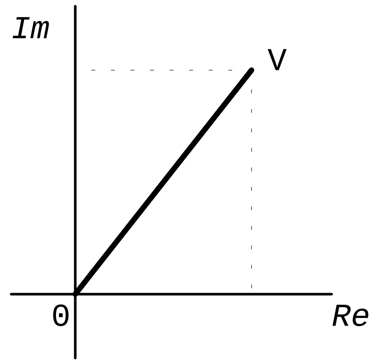
$$\therefore \tilde{I} = \frac{(5 + j0) - (5.(-0.5) + j.5.(-0.866))}{1} = \frac{5 + j.0 + 2.5 + j4.33}{1} = \frac{7.5 + j4.33}{1}$$

$$\therefore \tilde{I} = 8.65\angle 30$$

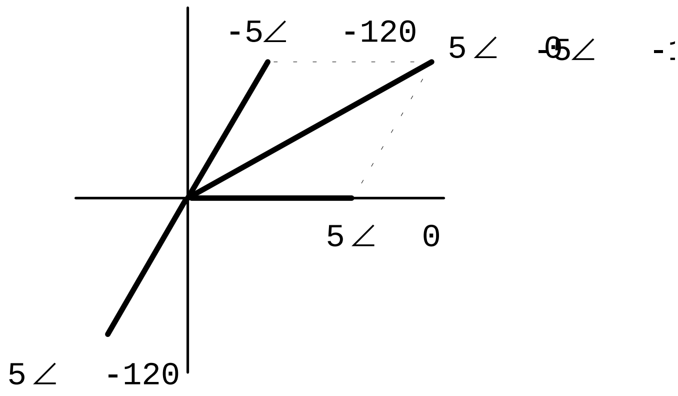
so the current i has a (peak) value of 8.65A and leads the reference voltage source by 30°: $i = 8.65.\cos(\omega t + 30^o)$

Phasor diagrams

are a way of representing the magnitudes and relative phase angles in a graphical format:



and can be used to check that a calculated answer is 'reasonable', as in the previous example

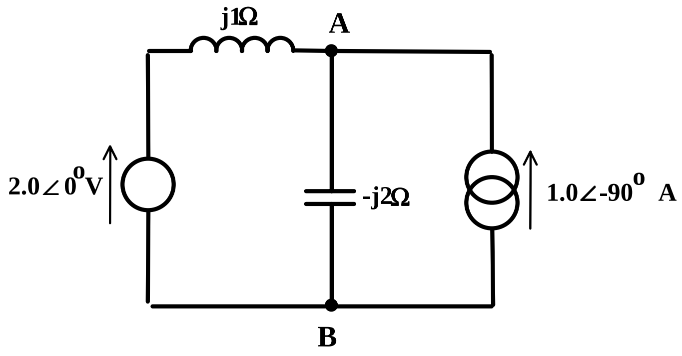


General ac circuit analysis

Techniques of circuit analysis (Nodal Voltage and Mesh Current) and network theorems (Thevenin, Norton, Superposition, Source Equivalence, Star/Delta Transformations) can be applied to ac circuits using phasor quantities and complex impedances.

Example

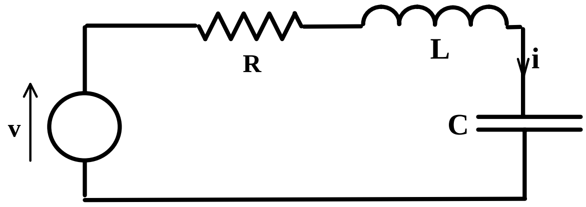
The following circuit can be solved by Nodal Voltage Analysis, Mesh Current Analysis, Source Equivalence or Superposition:



Resonant Circuits

An important group of ac circuits have the general property of behaviour which is very frequency sensitive. These are the resonant circuits, which have widespread applications in frequency selection.

Series resonance (LCR circuit)



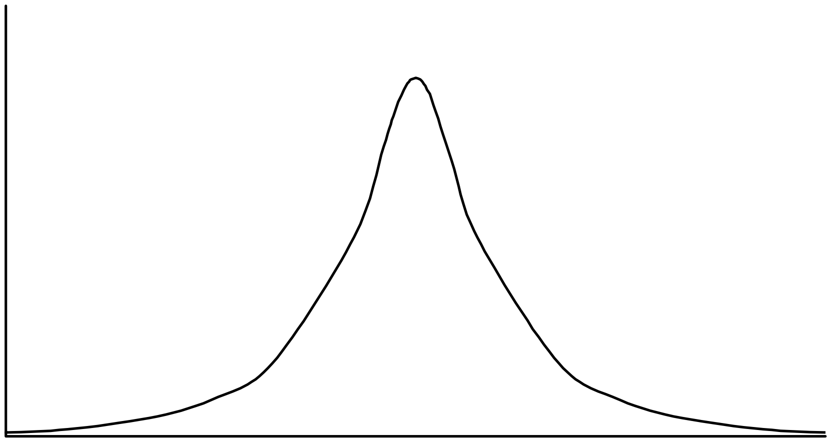
if:

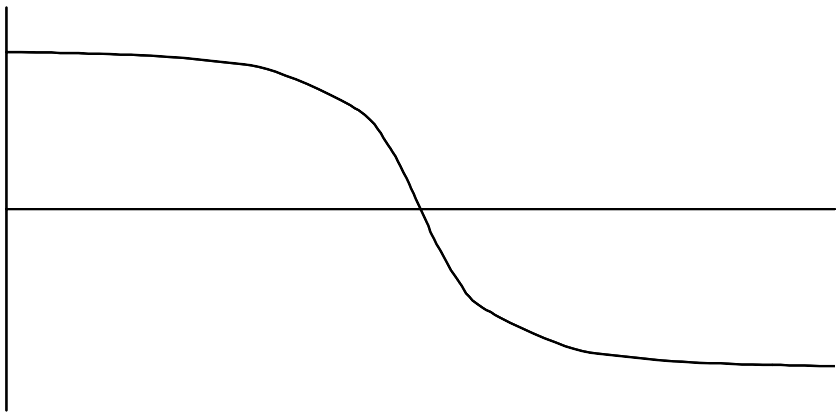
$$i = \tilde{I}e^{j\omega t} \ldots\ldots v = \tilde{V}e^{j\omega t}$$

$$\therefore \frac{\tilde{V}}{\tilde{I}} = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

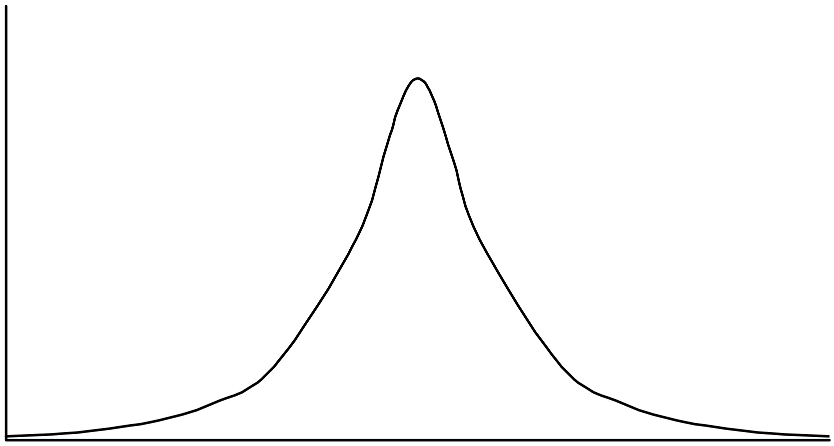
$$\therefore \tilde{I} = \frac{\tilde{V}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

for a voltage which is fixed in magnitude, but variable in frequency:





Half-power bandwidth and Q-factor



The angular frequencies where the current magnitude has fallen to 1/√2 of its peak value (V/R) are given by solutions of:

$$\left|\tilde{I}\right| = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}.R} \qquad \therefore 2R^2 = R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2$$

$$\therefore \pm R = \omega L - \frac{1}{\omega C} \qquad \therefore \omega^2.LC \pm \omega.CR - 1 = 0$$

$$\therefore \omega = \frac{\pm CR \pm \sqrt{(CR)^2 + 4LC}}{2LC}$$

$$\therefore \omega_2 = \frac{+\sqrt{(CR)^2 + 4LC} + CR}{2LC} \qquad \therefore \omega_1 = \frac{+\sqrt{(CR)^2 + 4LC} - CR}{2LC}$$

Define half-power bandwidth as the difference between the two angular frequencies at which the current magnitude is 1/√2 of its peak value

$$\therefore \omega_2 - \omega_1 = \frac{+\sqrt{(CR)^2 + 4LC} + CR}{2LC} - \frac{+\sqrt{(CR)^2 + 4LC} - CR}{2LC} = \frac{2CR}{2LC} = \frac{R}{L}$$

It is more useful to relate the half-power bandwidth to the resonant angular frequency, by means of the quality factor, Q:

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$$

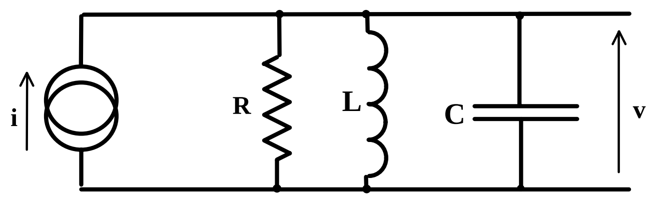
A circuit with a large Q-factor has a sharp resonant peak and is effective as a frequency selection circuit.

Note:

At the resonant frequency, ω_0, the capacitor voltage is:

$$\tilde{V}_c = \frac{1}{j\omega_0 C} \cdot \frac{V}{R} = -j\left(\frac{1}{\omega_0 CR}\right)V = -jQV$$

Parallel resonance (LCR circuit)



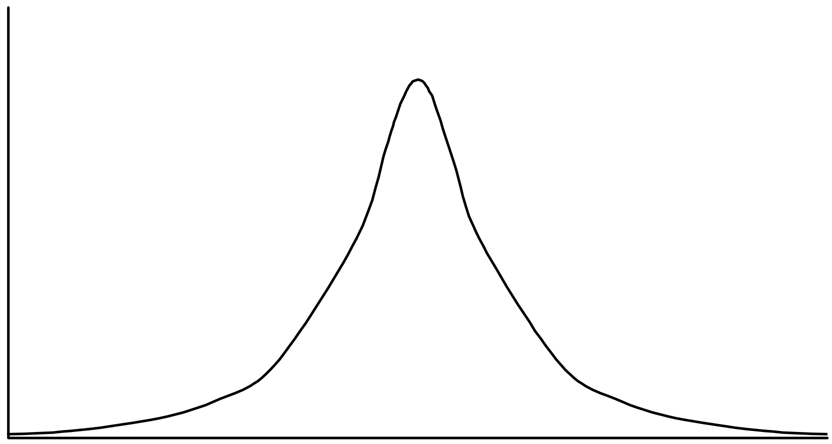
if:

$$i = \tilde{I}e^{j\omega t} \ldots\ldots v = \tilde{V}e^{j\omega t}$$

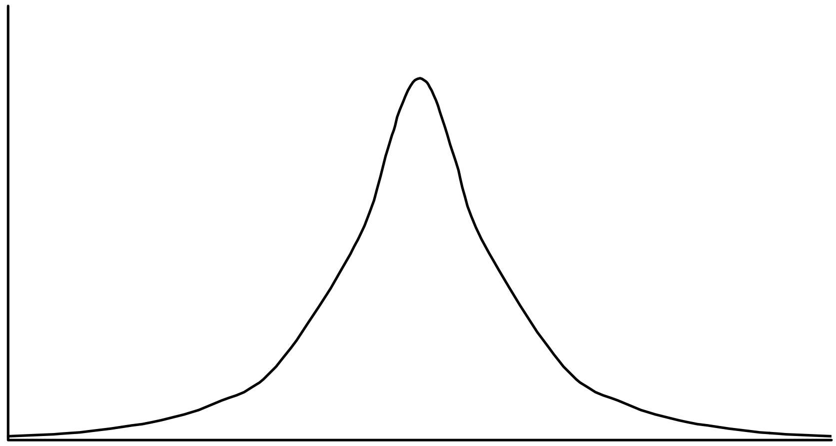
$$\therefore \frac{\tilde{I}}{\tilde{V}} = \frac{1}{R} + j\omega C + \frac{1}{j\omega L} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

$$\therefore \tilde{V} = \frac{\tilde{I}}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}$$

for a current which is fixed in magnitude, but variable in frequency:



Half-power bandwidth and Q-factor



Half-power bandwidth:

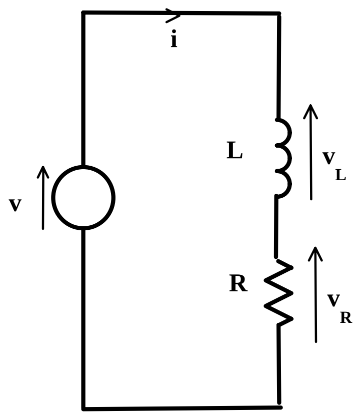
$$\therefore \omega_2 - \omega_1 = \frac{1}{CR}$$

and the quality factor:

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \omega_0 CR = \frac{R}{\omega_0 L}$$

Complex Power

Consider an inductance-resistance series connection:



if $v = \sqrt{2}.V.\sin(\omega t)$ and $i = \sqrt{2}.I.\sin(\omega t - \theta)$

so V and I are rms values, related by $I = V / \sqrt{R^2 + \omega^2 L^2}$, and $\theta = \tan^{-1}(\omega L / R)$

then the instantaneous voltages across the resistance and inductance can be written:

$$v_R = Ri = R.\sqrt{2}.I.\sin(\omega t - \theta) = \sqrt{2}.V_R.\sin(\omega t - \theta)$$

$$v_L = L.\frac{di}{dt} = \omega L.\sqrt{2}.I.\cos(\omega t - \theta) = \sqrt{2}.V_L.\cos(\omega t - \theta)$$

Instantaneous power to resistance, R =

$$v_R i = 2.V_R.I.\sin^2(\omega t - \theta) = V_R.I - V_R.I\cos 2(\omega t - \theta)$$

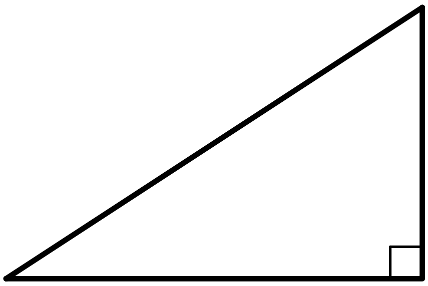
so the average power dissipated in the resistance =

Instantaneous power to inductance, L =

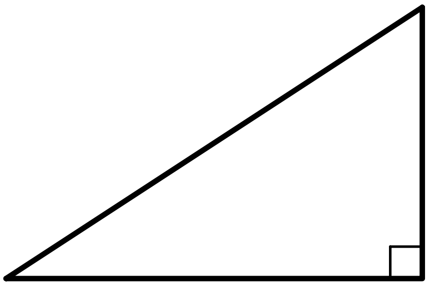
$$v_L i = 2.V_L.I.\cos(\omega t - \theta)\sin(\omega t - \theta) = 2V_L.I\sin 2(\omega t - \theta)$$

so the average power transferred to the inductance =

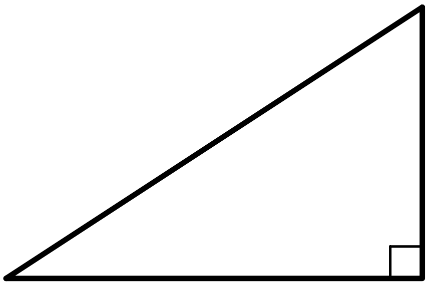
Phasor diagram:



Impedance diagram:



Power diagram:



Power = P = V_R.I = VI. cos(θ) unit: Watts

Reactive Power = Q = V_L.I = VI. sin(θ) unit: Volt.Amps reactive (Vars)

$(VI)^2 = P^2 + Q^2$

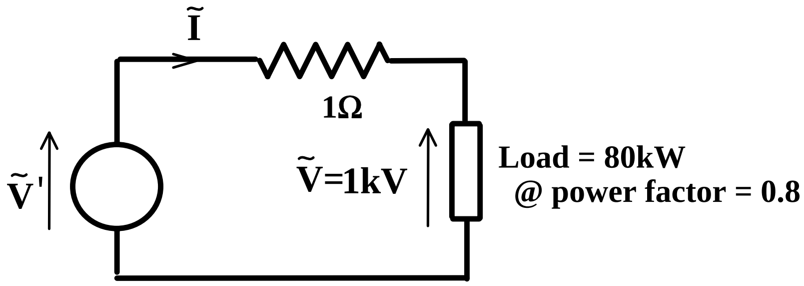
Principle of Conservation of Watts and Vars (Power and Reactive Power)

In a circuit the power and reactive power are separately conserved: the power input by the sources is absorbed by the resistors and the reactive power input is absorbed by the inductors and capacitors.

Note: Volt.Amps are *not* conserved.

Example

Find the voltage required to feed a load through cables of finite resistance:



At the load:

P =

I =

V.I =

sin θ =

Q =

Power dissipated in cable =

Using Conservation of Power and Reactive Power, calculate supply conditions

P'=

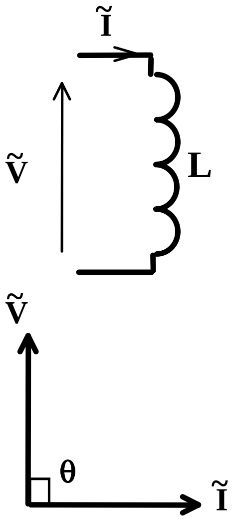
Q' =

$$(V'I')^2 = P'^2 + Q'^2$$

$$\therefore V' = \frac{\sqrt{P'^2 + Q'^2}}{I'} =$$

In this example 10kW of power is wasted because a current of 10A flows through the 1Ω resistance. Can the current be reduced?

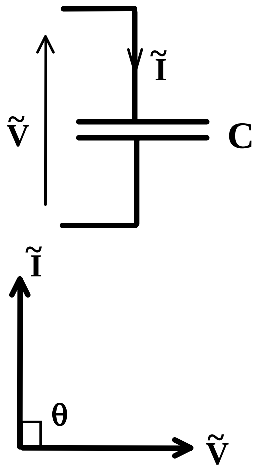
Reactive power in inductances and capacitances



$$Q = V.I.\sin\theta$$

if θ is positive for $\tilde{I}$ lagging $\tilde{V}$ and θ =90°, V=ωLI:

$$Q_L = \omega L I^2 = \frac{V^2}{\omega L}$$



$$Q = V.I.\sin\theta$$

if θ is negative for $\tilde{I}$ leading $\tilde{V}$ and θ =-90°, V=I/ωC:

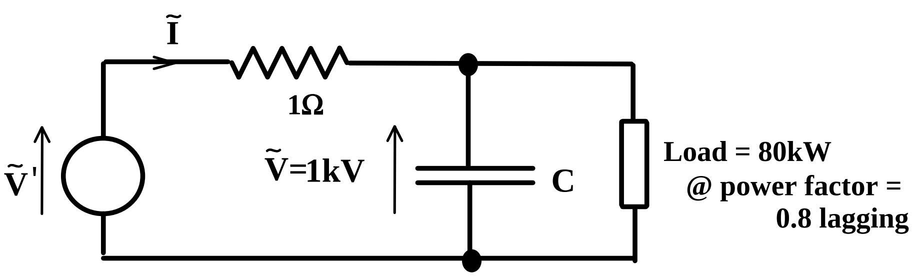
$$Q_C = -\omega C V^2 = -\frac{I^2}{\omega C}$$

so the capacitor absorbs negative reactive power and the inductor absorbs positive reactive power.

inductive circuits having a lagging power factor (current lags voltage)
capacitive circuits having a leading power factor (current leads voltage)

Power Factor Correction

A combination of capacitors and inductors may absorb net reactive power = 0



Find C to minimise the supply current:

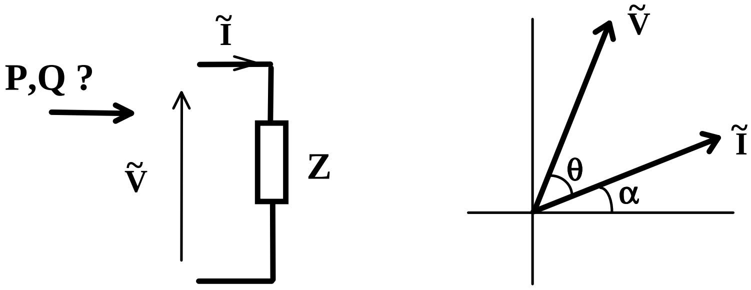
Load absorbs 60kVAr, so choose C to absorb -60kVAr:

$$-60\times10^3 = -\omega CV^2 = -2\pi\times50\times C\times\left(10^3\right)^2$$

$$\therefore C = \frac{60\times10^3}{2\pi\times50\times\left(10^3\right)^2}F = 191\mu F$$

load and capacitor combined then absorb 80kW power only, so I=80A and
cable loss = 6.4kW

Complex power in terms of phasors



$$\tilde{V} = V\angle\theta+\alpha \qquad \tilde{I} = I\angle\alpha$$

consider:

$$\tilde{V}.\tilde{I} = V\angle\theta+\alpha\times I\angle\alpha = VI\angle\theta+2\alpha = VI\cos(\theta+2\alpha)+jVI\sin(\theta+2\alpha)$$

and:

$$\tilde{V}.\tilde{I}^* = V\angle(\theta+\alpha)\times I\angle-\alpha = VI\angle(\theta) = VI\cos(\theta)+jVI\sin(\theta)$$

$$\therefore S = \tilde{V}.\tilde{I}^* = P+jQ$$

where S is the **complex power**

Tutorial 1: Dc Circuits I

1. Find the current I in each of the circuits in Fig. 1.

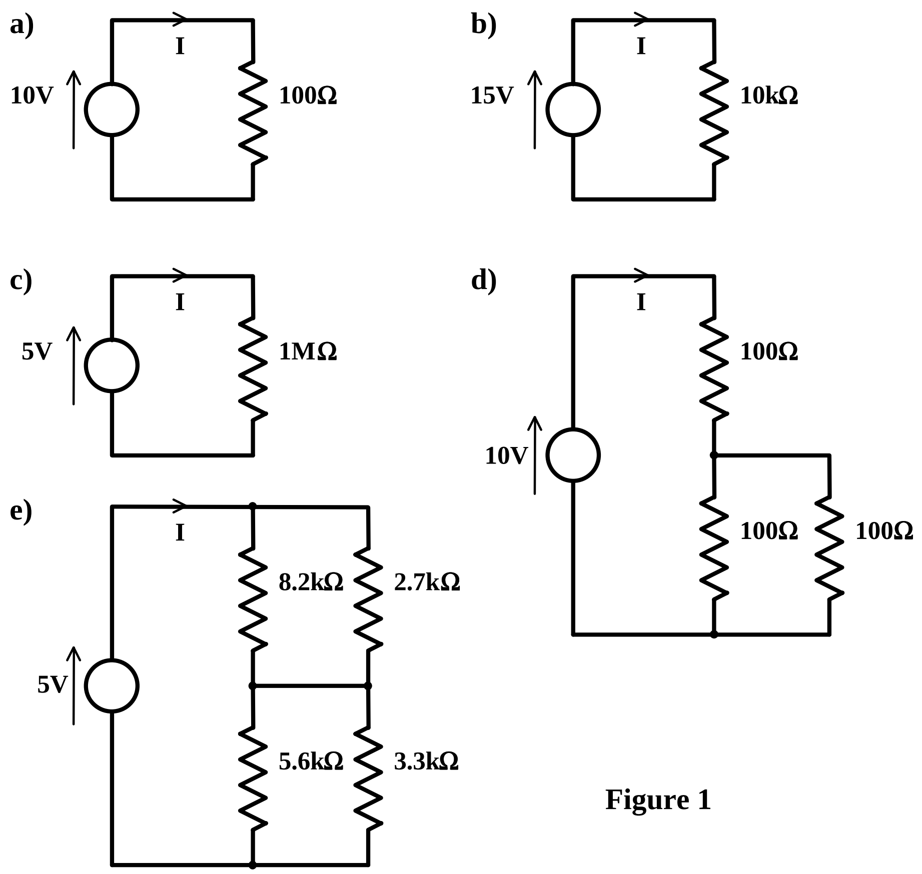


Figure 1

2. Find the voltage, V, in the circuit of Fig. 2 if the value of the resistance R is: a) 1MΩ; b) 100kΩ; c) 10kΩ; d) 1kΩ; e) 100Ω.

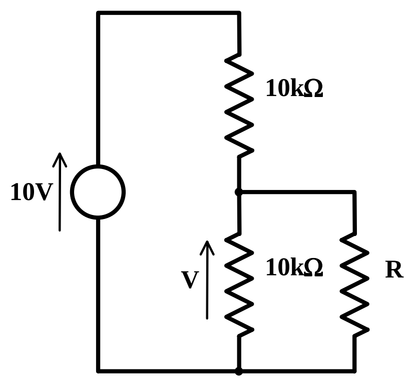


Figure 2

3. For each of the circuits in Figure 3, write down the voltage V and current I:

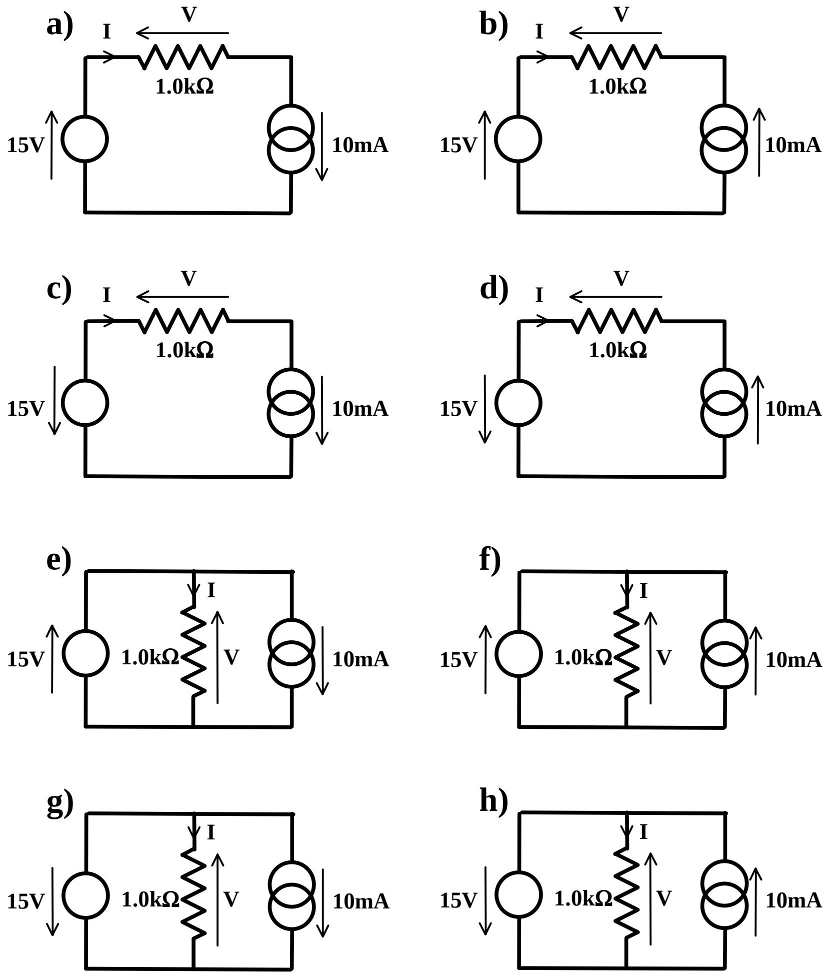


Figure 3

4. a) The electrical characteristics of a battery were investigated using the circuit shown in Fig. 4a. Values of V and I were recorded as the load resistance R was varied, and the following results were obtained:

V(v)	12.0	11.5	11.0	10.5	10.0
I(mA)	0	50	100	150	200

Show that the battery can be modelled by either of the circuits in Figs 4b and 4c, and calculate the values of V_T, I_N, and R.

b) Two identical batteries are connected in parallel, with terminals of like polarity together, as shown in Fig. 4d. Calculate the terminal voltage V, for the following values of load resistance R_L: i) 10Ω; ii) 15Ω; iii) 25Ω.

c) For each value of load resistance in part b) calculate the current supplied by each battery.

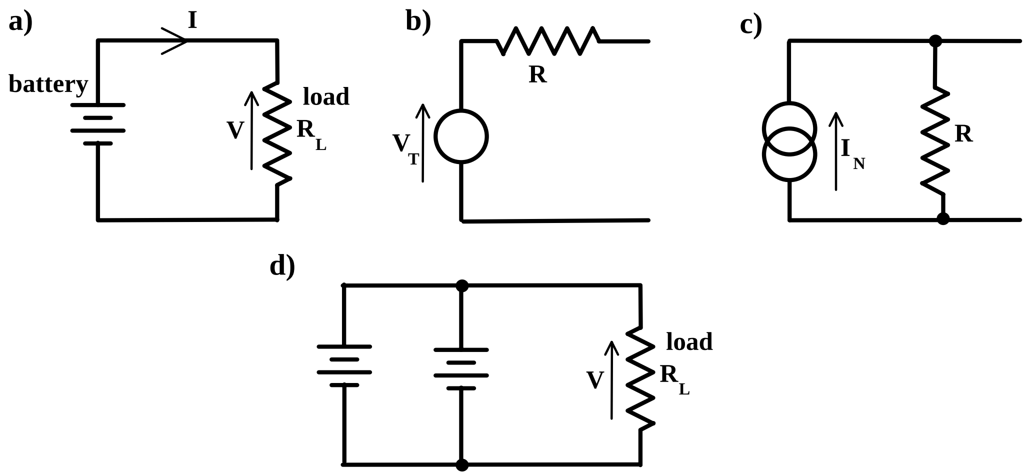


Figure 4

5. Solve (i.e. find the voltage across and current through each element) the circuits shown in Fig. 5. Check that your answers are correct using Kirchoff's Laws.

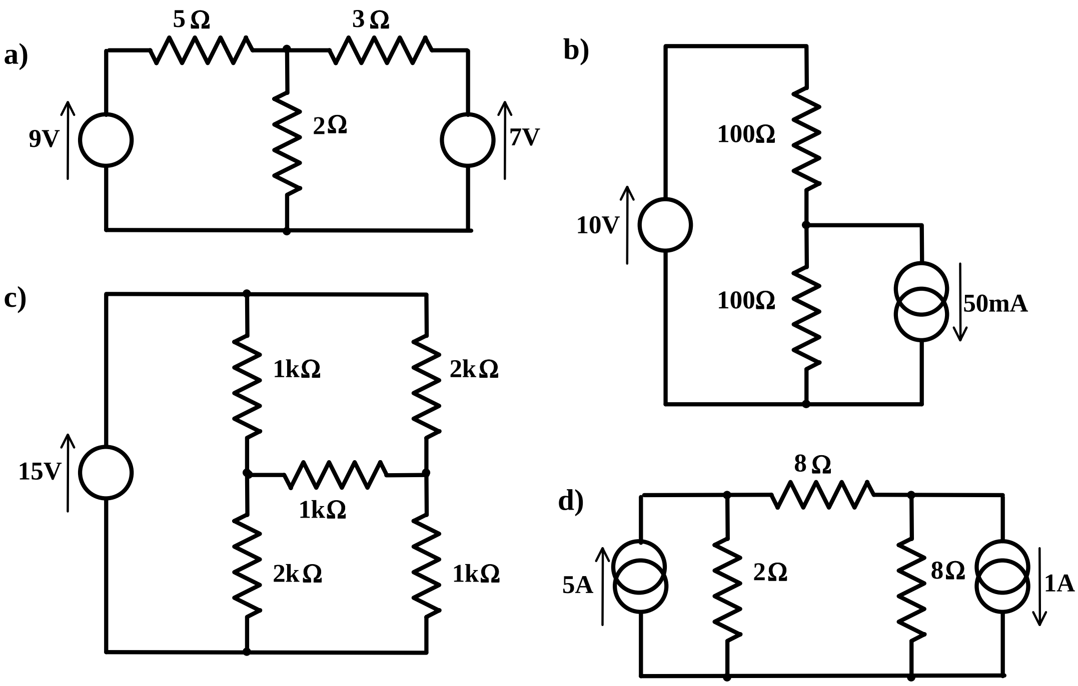


Figure 5

Answers:

1. a) 0.1A; b) 1.5mA; c) 5µA; d) 67mA; e) 1.22mA
2. a) 4.98V; b) 4.76V; c) 3.33V; d) 0.83V; e) 0.1V
3. a) 10mA, 10V; b) –10mA, -10V; c) 10mA, 10V; d) –10mA, -10V; e) 15V, 15mA; f) 15V, 15mA; g) –15V, -15mA; h) -15V, -15mA
4. a)12V; 1.2A; 10Ω; b)i) 8V; ii) 9V; iii) 10V; c) i) 0.4A; ii) 0.3A; iii) 0.2A
5. Check them for yourself!

Tutorial 2: Dc Circuits II

1. Solve (i.e. find the voltage across and current through each element) the circuits shown in Fig. 1, using Mesh Current Analysis. Check that your answers are correct using Kirchoff's Laws.

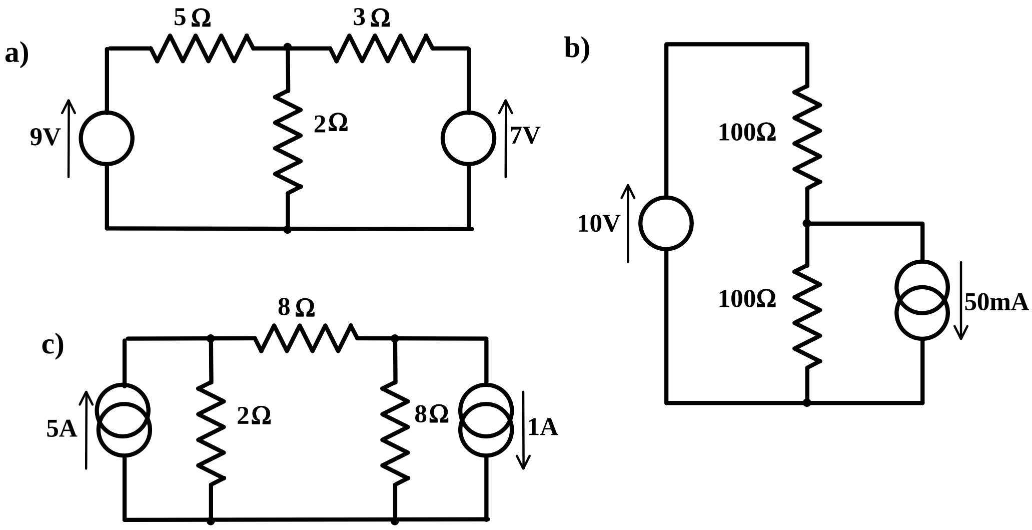


Figure 1

2. Find the Thevenin equivalent, between terminals A and B, of the circuit shown in Fig. 2.

What value of load resistance must be connected between terminals A and B in order to maximise the power delivered to the load, and what is this maximum power?

With the load power maximised, find the power output of the 15V source.

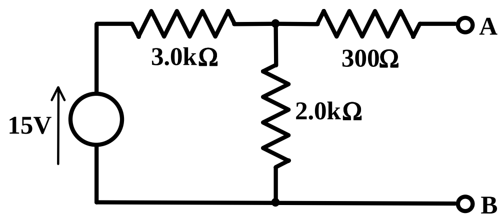


Figure 2

3. Using the Superposition Theorem, calculate the current I in the network of Fig. 3.

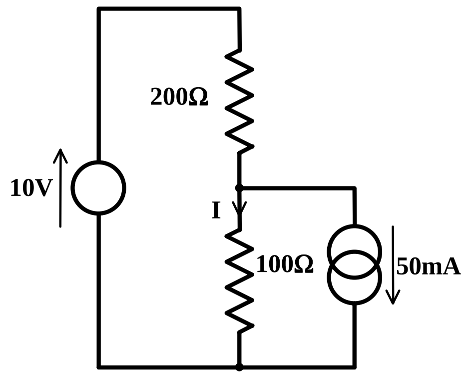


Figure 3

4. Use the delta-star transformation to solve the circuit of Fig.4.

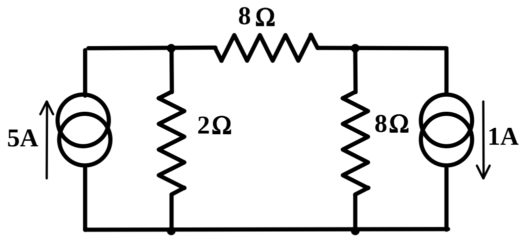


Figure 4

Answers:

1. Check answers yourself. **2.** 6V; 1.5kΩ; 1.5kΩ; 57mW. **3.** 0mA. **4.** 4A, 1A, 0A.

Tutorial 3: Transients

1. Calculate the time constant for changes in currents and voltages following operation of the switch S in each of the circuits in Fig. 1.

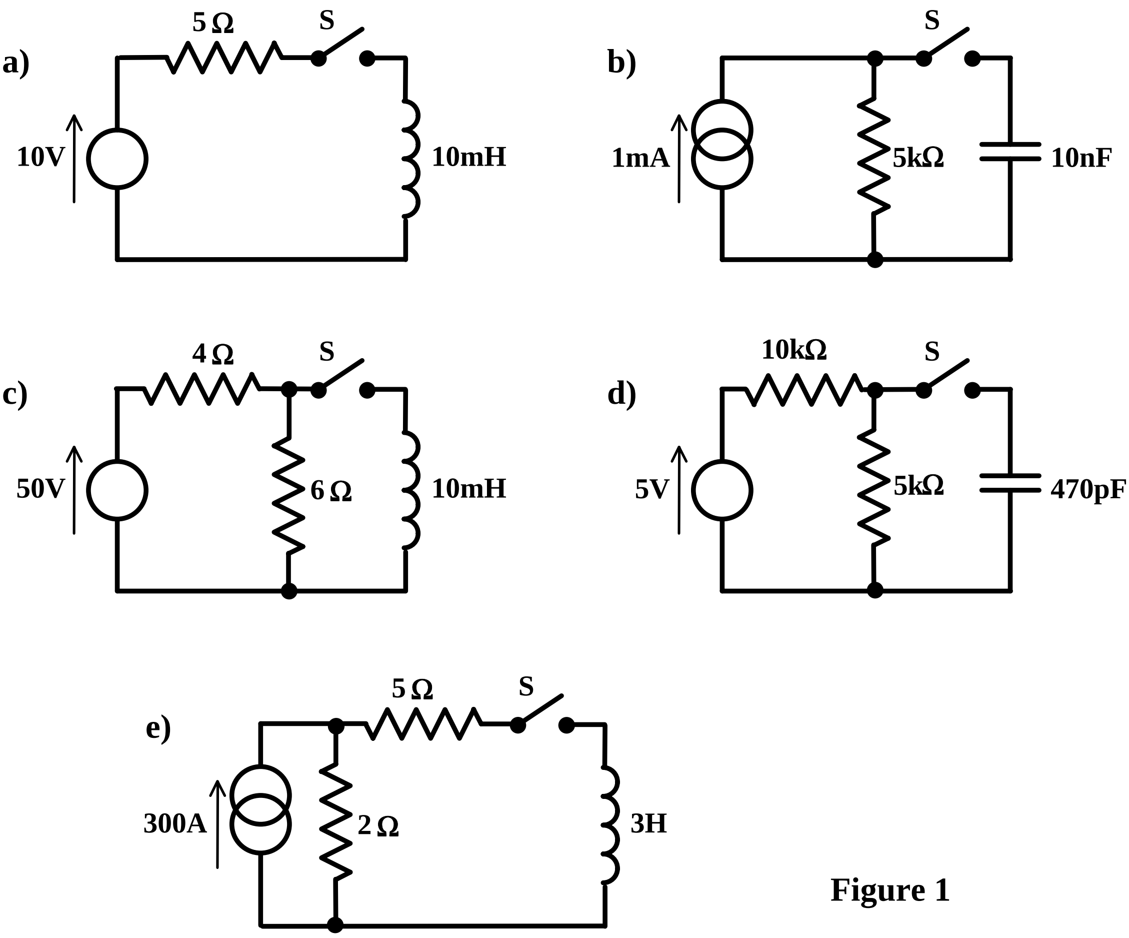


Figure 1

2. In the circuit of Fig. 2, the capacitor is initially uncharged. The switch S closes at time t=0. Calculate the values of v_C, i_C, i_r at the following times: i) t=20µs; ii) t=50µs; iii) t=100µs.

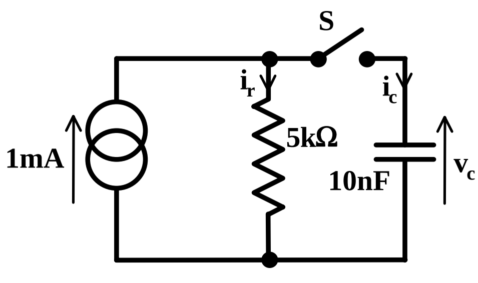


Figure 2

3. In the circuit of Fig. 3, the switch S closes at time t=0. Calculate the values of v_L and i at the following times: i) t=1ms; ii) t=2ms; iii) t=5ms.

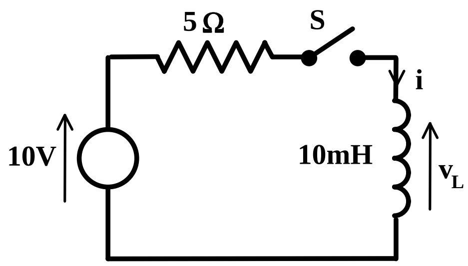


Figure 3

4. a) In the circuit of Fig. 4, calculate the steady-state value of the voltage across the capacitor v_C when the switch S is i) open and ii) closed.

b) Calculate the time constant for changes in v_C after the switch S: i) opens and ii) closes.

c) The circuit is in the steady-state with switch S open. If the switch S is closed at time t=0, calculate the value of v_C when t=1µs.

d) The circuit is in the steady-state with switch S closed. If the switch S is opened at time t=0, calculate the value of v_C when t=1µs.

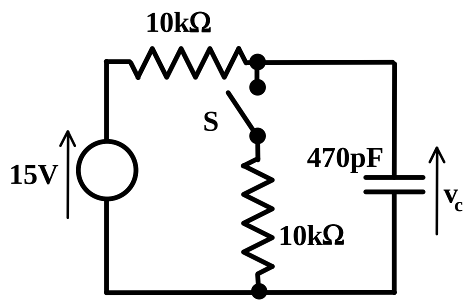


Figure 4

5. If the circuit of Fig. 5 is in the steady-state with the switch S closed, calculate i) the energy stored in the capacitor, and ii) the power being dissipated in the 1MΩ resistor.

Calculate the time constant for the decay of the capacitor voltage after the switch is opened.

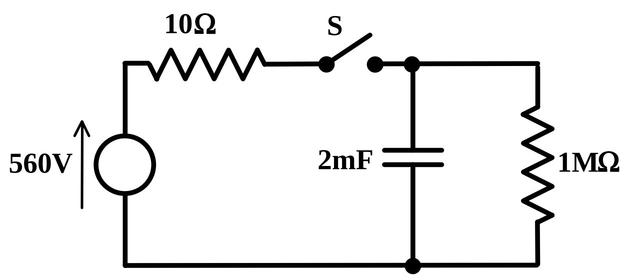


Figure 5

6. If the circuit of Fig. 6 is in the steady-state with the switch S closed, calculate the energy stored in the inductor.

Explain the function of the diode D and calculate the peak current which flows in it.

Calculate the time constant for the decay of inductor current after the switch S is opened, assuming that the diode is ideal.

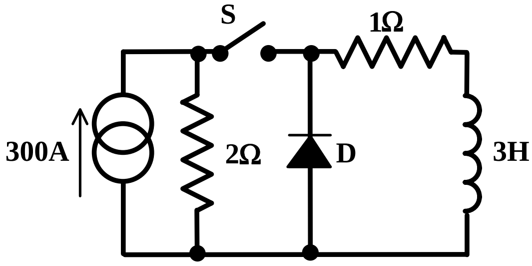


Figure 6

Answers:

1. a) 2ms; b) 50µs; c) 4.2ms; d) 1.57µs; e) 0.42s.
2. i) 1.65V; 0.67mA; 0.33mA ii) 3.16V; 0.368mA; 0.632mA iii) 4.33V; 0.135mA; 0.865mA.
3. i) 6.07V; 0.79A ii) 3.68V; 1.26A iii) 0.82V; 1.84A.
4. a) i) 15V ii) 7.5V b) i) 4.7µs ii) 2.3µs c) 12.4V d) 8.9V
5. i) 314J; ii) 0.31W; iii) 2000s. 6. 60kJ; 200A; 3s.

Tutorial 4: Ac circuits

1. In the circuits of Fig. 1 a-d calculate the phasor voltage **V** and in the circuits of Fig. 1 e-h calculate the phasor current **I**.

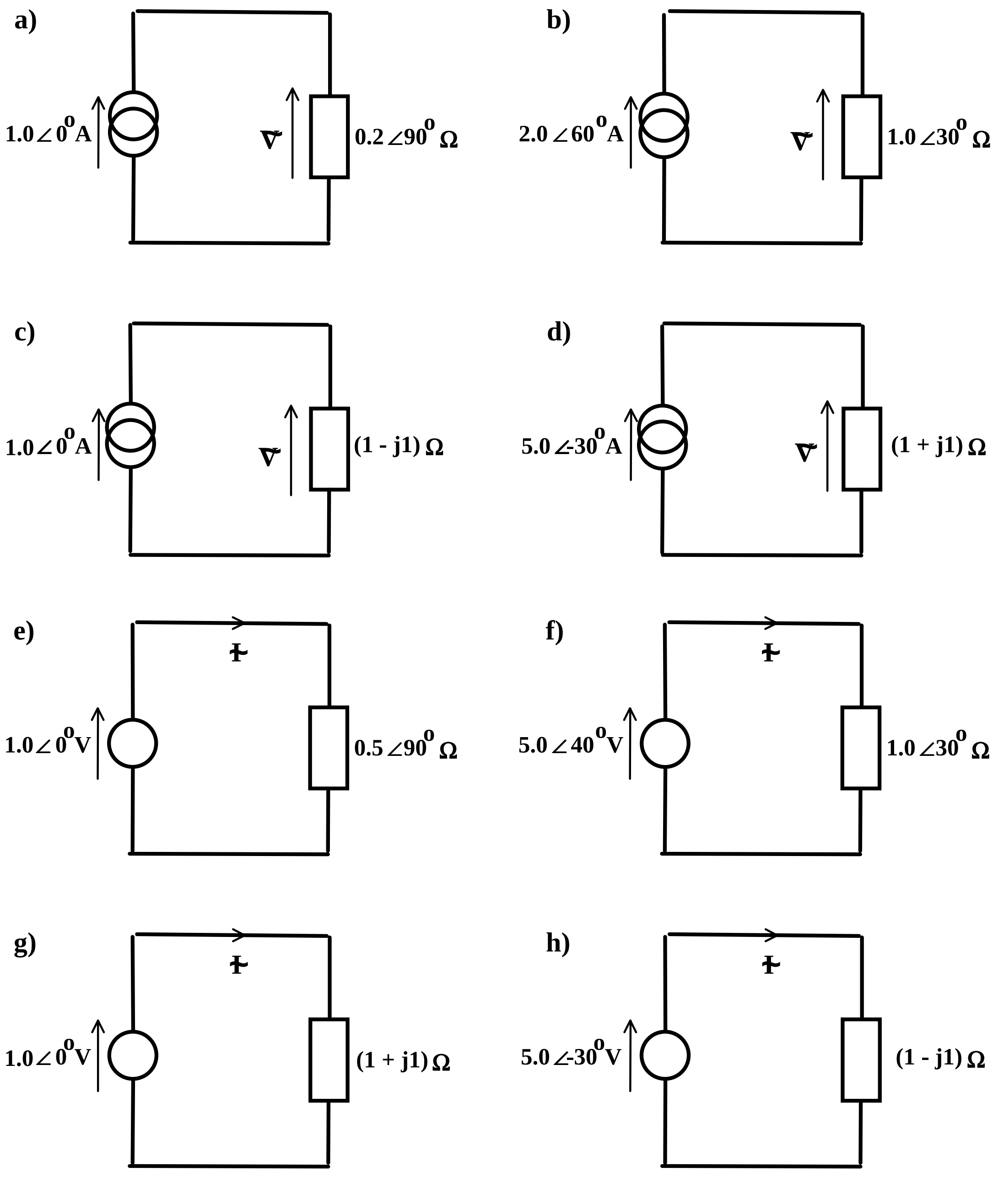


Figure 1

2. For each of the circuits shown in Fig. 2, calculate the complex impedance at a frequency of 10kHz:

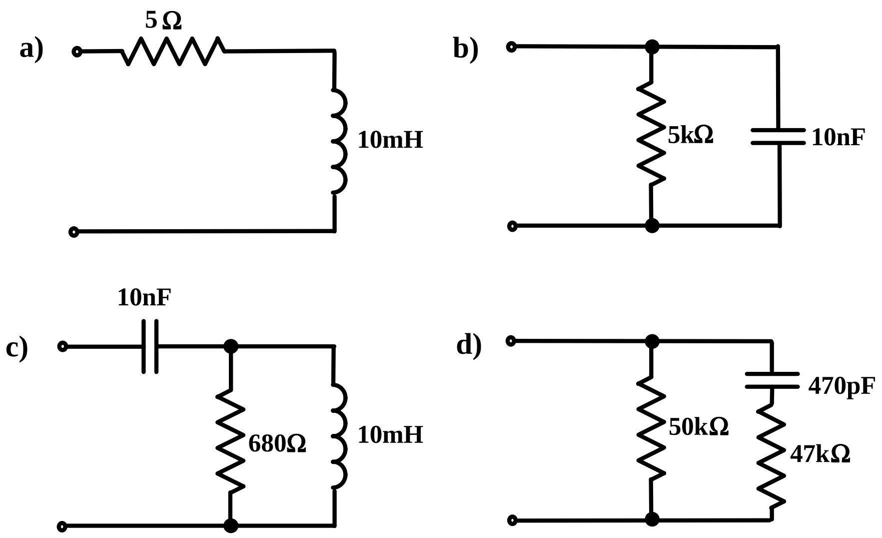


Figure 2

3. A ‘balanced three phase set of voltages’ consists of three voltages with equal magnitude and phase differenceof 120°. So in Fig 3, the voltages V_{ao}, V_{bo}, V_{co} are a balanced three phase set. Show that the voltages V_{ab}, V_{bc}, V_{ca} also form a balanced three-phase set.

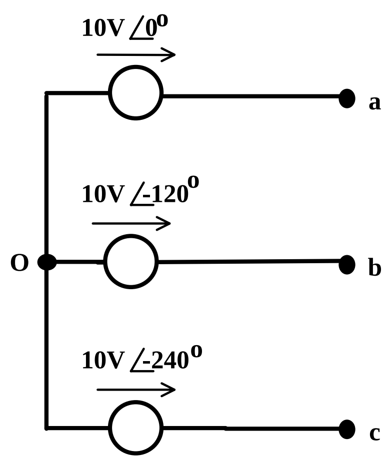


Figure 3

4.
The networks shown in Figure 4 are to be solved using Nodal Voltage Analysis. For each network:

a) Define the phasor voltage between the nodes A-B as $\tilde{V}$.

b) Write down expressions for the current in each of the circuit elements in terms of the voltage $\dot{V}$.

c) Using Kirchoff’s Voltage Law at one of the nodes, A or B, derive an equation relating the currents flowing in the branches connected to that node and solve it to find $\dot{V}$.

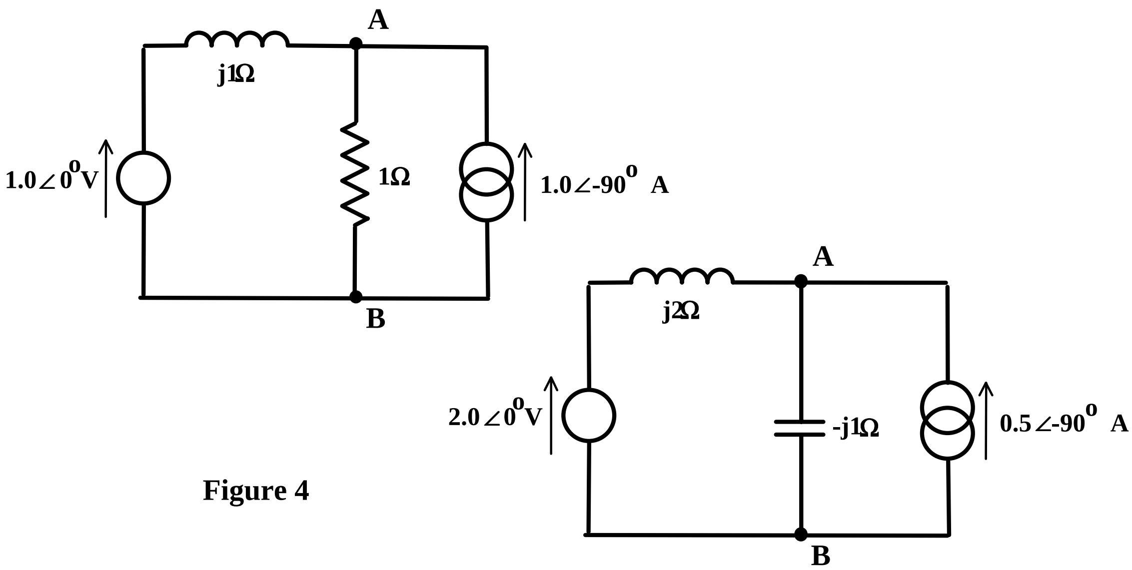


Figure 4

5. A series resonant (L-C-R) circuit is driven by an ideal voltage source and has L = 10mH, C = 10nF, R = 20Ω. Calculate the resonant frequency and Q-factor of the circuit.

6. Three loads are partially specified in the following table:

	kVA	kW	kVAr	power factor
Load 1	250			0.5 lagging
Load 2		180		0.8 leading
Load 3	300		100 lagging	

Complete the table.

If the three loads are connected in parallel to a 2kV, 50Hz supply, what is the resultant power factor? What value of capacitor must be connected in parallel to give a resultant power factor of unity?

Answers:

1. a) $0.2 \angle 90^{o}$ V; b) $2.0 \angle 90^{o}$ V; c) $1.41 \angle -45^{o}$ V; d) $7.0 \angle 15^{o}$ V;
   e) $2.0 \angle -90^{o}$ A; f) $5.0 \angle 10^{o}$ A; g) $0.71 \angle -45^{o}$ A; h) $3.54 \angle 15^{o}$ A.

2. a) $628 \angle 89.5^{o}$ Ω; b) $1.52 \angle -72^{o}$ kΩ; c) $1.29 \angle -76^{o}$ kΩ; d) $28.2 \angle -16.5^{o}$ kΩ;

3. V_{ab} = $17.3 \angle 30^{o}$ V; V_{bc} = $17.3 \angle -90^{o}$ V; V_{ca} = $17.3 \angle -210^{o}$ V.

4. $1.41 \angle -45^{o}$ V; $3.0 \angle 180^{o}$ V

5. 15.9kHz, Q= 50

6. 125kW, 216kVAr, 225kVA, -135kVAr, 283kW, 0.945, 0.955 lagging, 144μF